ESSAYS ON THE

DYNAMICS OF INSTITUTIONAL REFORM

by

Leyla Derin Karakaş

A dissertation submitted to Johns Hopkins University in conformity
with the requirements for the degree of Doctor of Philosophy

Baltimore, Maryland

July 2014

Abstract

This dissertation contributes to the knowledge on the emergence of political institutions related to checks and balances as well as their effects on economic outcomes. The first two chapters analyze the determinants of constitutional limits on executive authority and the implication of these limits for policy-making. The third chapter studies the process by which institutional reforms are enacted.

While the past century witnessed a gradual adoption of limits on executive power, countries that oscillate between periods of strong and weak executive constraint-regimes still exist. The literature explains the emergence of strong political institutions that effectively curb the power of the executive with competitive electoral races. However, empirical evidence indicates that governments facing similar electoral pressures have made widely ranging institutional decisions on this issue. This observation suggests that factors besides a country's degree of electoral competitiveness must be influencing the extent of its reforms.

To shed light on this discrepancy between the theoretical literature and the data, the first chapter of this dissertation constructs a dynamic model of political competition in which limits on executive decision-making that will constrain the future government are chosen by the current party in power. The basic results affirm an incumbent's main trade-off identified in the literature: While loose executive constraints permit an incumbent to enact his desired policies in case of re-election, the same would apply to his opponent under the opposite scenario. This chapter's contribution is to show that this trade-off is not constant. Specifically, the incentives that

shape an incumbent's institutional decision evolve with his country's level of public sector development. The analysis suggests that this evolution is due to governments being more inclined to make common cause with their opponents when there exist mutually beneficial gains to be realized from public sector investments such as infrastructure spending. In these circumstances, the common cause motivation dominates the inherent conflict between parties over unproductive political spending. Consequently, executive constraints would initially be kept looser in order to enable such investments.

The main results corroborate the empirical evidence by showing that higher levels of public sector development in a country will be associated with tighter constraints on the executive branch. Moreover, these tighter constraints will be less sensitive to swings in political power. While these results confirm the importance of the degree of electoral competitiveness in determining institutional outcomes, they offer an important qualification: The role played by elections depends on the country's public sector development. In addition, this chapter finds that public goods will be underprovided, even when political parties share the same preferences over it, due to the ever-present motive to restrict an opponent's political spending through institutional design. Overall, this chapter offers an explanation for the observed trend in executive constraints by focusing simultaneously on its relationship to indicators of electoral competitiveness and of public good provision.

Generalizing the framework employed in the first chapter, the second chapter of this dissertation focuses on the broader question of why democratic regimes tend to persist once they are established. The model of political competition developed in this chapter features an incumbent who can make reversible investments into a future government's ability to reform constraints on the executive branch. Examples of these investments include strengthening of press freedoms or of judicial independence. The main results suggest that polarization between the political parties and the competitiveness of elections lead to high and persistent levels of such investments. These higher costs of institutional reform in turn result in durable strong executive constraint-regimes.

The final chapter turns its attention to the process of institutional reform by analyzing within a bargaining framework the effect of a referendum option on the reform proposals already passed in the legislature. The findings indicate that surplus coalitions may be observed even though smaller coalitions would be sufficient for passage. An important result is that disparities in post-bargaining power such as campaigning resources incentivize challenge procedures to the detriment of grand bargains. Moreover, when achieved, such a grand bargain empowers the smaller parties in the legislature through favorable provisions in the reform bill. These results carry potential policy implications for forms of post-bargaining power during referendum campaigns, such as caps on campaign contributions.

Keywords : Endogenous political institutions, Executive constraints, Public goods, Pork-barrel spending, Bargaining, Referendum campaigns.

JEL Classification : C73, D72, D78, H11, H41, H42

Primary Reader : Hülya K. K. Eraslan

Secondary Reader : M. Ali Khan

Acknowledgements

I am grateful to my advisor Hülya Eraslan for her invaluable guidance and support, and to Ali Khan for his guidance and many stimulating conversations. I would also like to thank the seminar participants at Johns Hopkins University, The 23rd Summer Festival on Game Theory in Stony Brook, 2013 Georgetown Center for Economic Research Biennial Conference, 13th International Meeting of the Association for Public Economic Theory, and the 2013 Asian Meetings of the Econometric Society. All errors are mine.

Dedication

This dissertation is dedicated to my parents, Işıl and Eser Karakaş, who instilled in me the indispensable value of hard work while showing the things in life that are worth taking a break for.

Contents

List of Figures

Chapter 1

Institutional Constraints and Dynamic Inefficiency of Public Investments

1.1 Introduction

Political institutions that impose strong constraints on executive authority are an important feature of democracies. Examples of such institutions include legislative rules that allow for a fair representation of opposition political parties, strong protection of minority rights, or an independent judiciary that enforces constitutional principles. In this chapter, I focus on constitutional limits on the actions of the executive as a representative institution of checks and balances.

I measure the extent of limits on executive authority by the executive constraints score constructed in the Polity IV Project's "Regime Authority Characteristics and Transitions Datasets". Using a seven-point scale, this dataset categorizes executive decision-making from representing "Unlimited authority" to "Executive parity or subordination".[1] Evidence in support of a country being characterized as an unlimited authority regime include frequent use of rule by decree or blatant violations of the constitution by the executive. At the other extreme end, regimes with executive subordination are described as having the executive branch heavily dependent on a legislature for decision-making.

Analyses of constitutional limits on executive authority based on Polity IV scores indicate that while the past century witnessed the gradual adoption of executive constraints, regimes that oscillate between periods of strong and weak scores still persist.[2] Based on a sample of countries that have been in existence since the beginning of the 20^{th} century, the data exhibits periodic fluctuations until the 1970s in the number of countries with the highest executive constraint scores. These fluctuations follow the historical democratization trends of the last century.[3] The mid-70s mark a break with this trend as the democratization movements start taking hold in parts of Latin America and Asia. Figure 1.1 is reproduced from Besley and Persson (2011) and summarizes these movements.

[1] Various data on political institutions of each country is collected by the Polity IV Project and is available at http://www.systemicpeace.org/inscrdata.html. The score of interest in this chapter is coded as "XCONST". In order of strengthening executive constraints, a country can be categorized as "Unlimited Authority", "Intermediate Category", "Slight to Moderate Limitation on Executive Authority", "Intermediate Category", "Substantial Limitations on Executive Authority", "Intermediate Category", or "Executive Parity or Subordination".

[2] See Besley and Persson (2011), Chapter 7, p. 261 for a discussion on the evolution of executive constraints.

[3] See Huntington (1991) and Hobsbawm (1994).

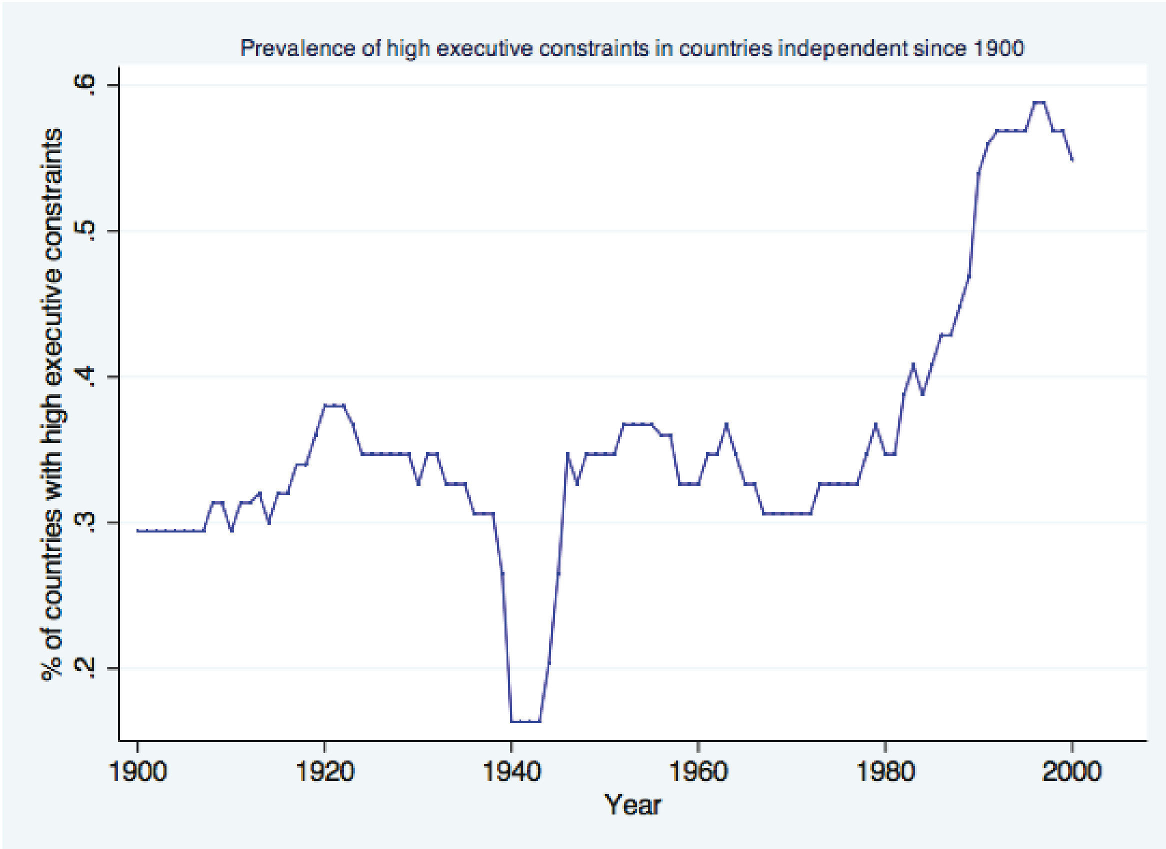

Figure 1.1: Prevalence of high executive constraints in countries independent since 1900.

The optimism created by the upward trend in Figure 1.1 towards strong executive constraints is shadowed once countries that have gained their independence after World War II are added to the sample. Among these countries, we observe that only a small fraction, including India, Israel and Botswana, have managed to successfully integrate strong checks and balances into their constitutional systems. The more common outcome is for the scores to fluctuate, which is observed in countries such as Sri Lanka, Turkey, and Indonesia. Figure 1.2 represents the evolution of average executive constraint scores over time based on the sample of newly-independent countries. It is easily observed that while these countries have shared in the historical democra-

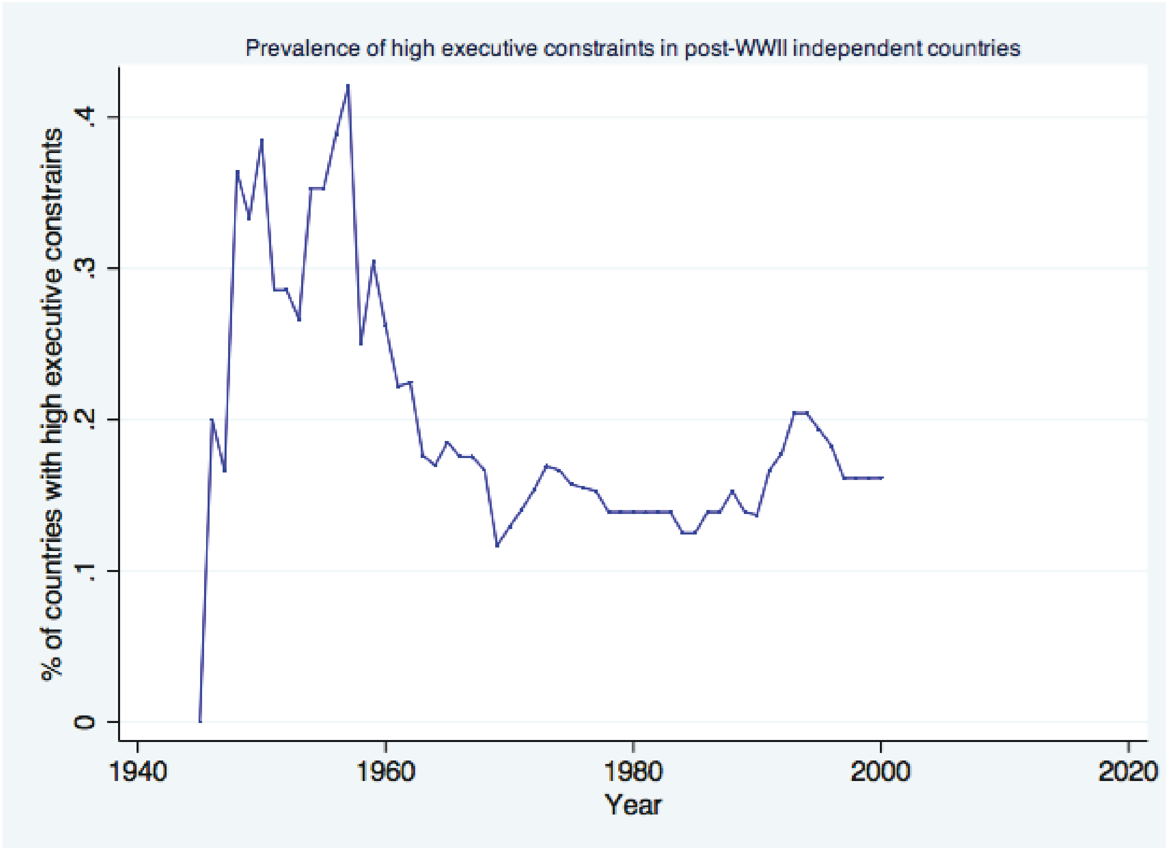

Figure 1.2: Prevalence of executive constraints in countries that have gained their independence after 1945.

tization movements, the upward trend towards stronger executive constraints is more muted. Figure 1.3 documents country-specific trends, focusing on Japan, Belgium, and Israel on the left column, and Argentina, Sri Lanka, and Turkey on the right column. While each of these countries has a unique history of economic development and political conflict, they demonstrate the different paths institutionally stable and unstable countries have taken. Specifically, Japan, Belgium, and Israel exhibit the stability of strong executive constraints in the post-war period, whereas Argentina, Sri Lanka, and Turkey represent cases of institutional instability.

This chapter studies the determinants of executive constraints with a focus on

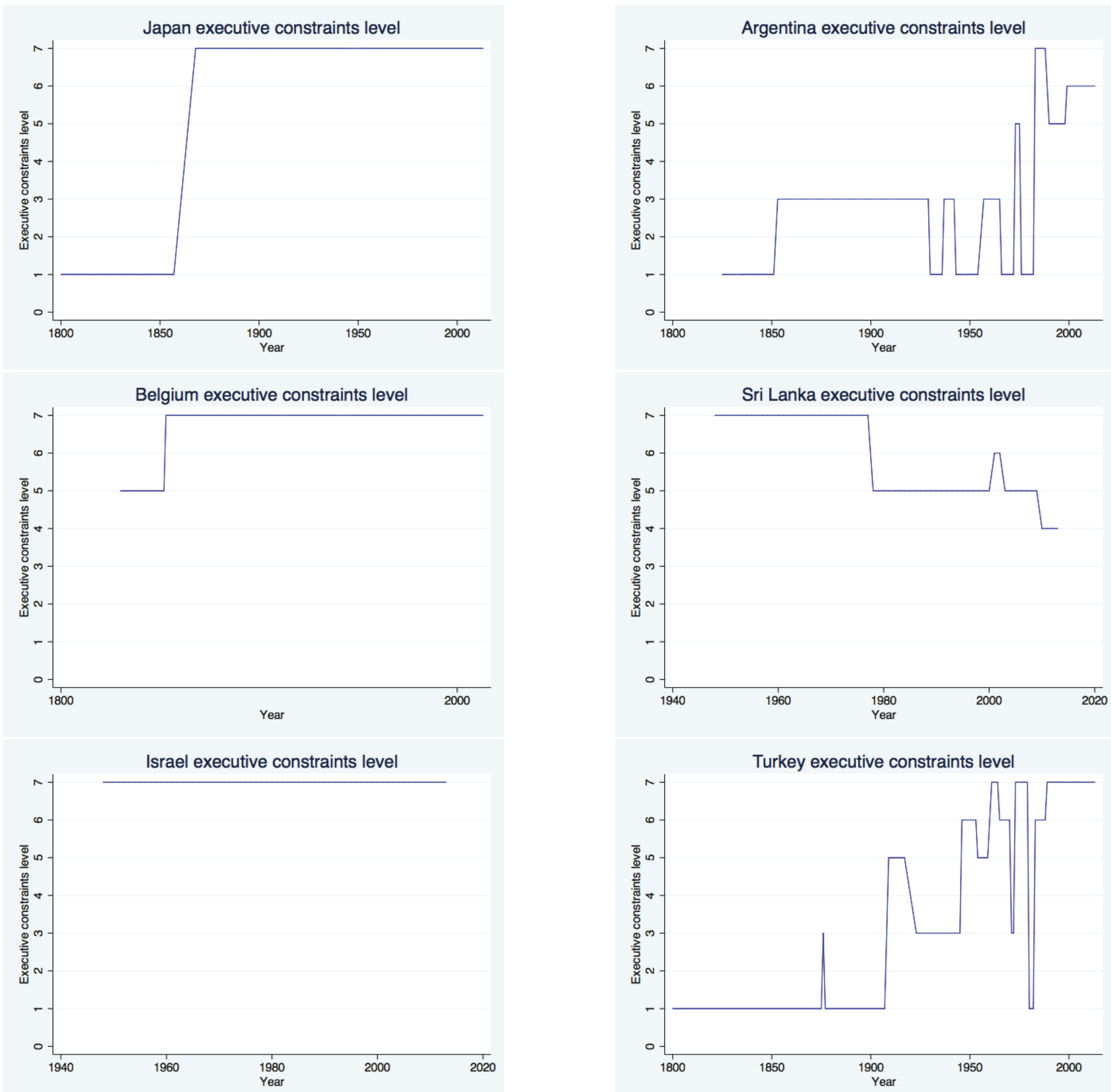

Figure 1.3: Examples of countries with stable versus unstable executive constraints.

understanding the factors that lead to the divergent experiences exemplified in Figure 1.3. The literature has predominantly linked the strength of a regime's executive constraints to its political turnover characteristics by arguing that leaders who are more likely to remain in power do not have an incentive to promote strong constraints on their rule.[4] However, these studies have not addressed the question of why electorally-powerful incumbents in democratic regimes do not weaken these constraints. For example, the Polity IV data indicates that the thirteen years of Labour governments between 1997 and 2010 have not resulted in a reversal of U.K's strong executive constraint scores. Likewise, consecutive governments led by Sweden's Social Democratic Party have not engaged in attempts to roll back the country's strong system of checks and balances. These observations suggest that factors besides electoral uncertainty must be influencing the institutional decisions of governments across regimes and resulting in different responses to similar political environments. Therefore, while middle regimes with oscillating executive constraint scores fit the predictions of the existing theoretical models, the resilience of strong institutions in the face of electoral advantage in established democracies poses a challenge to their results.[5]

The aim of this chapter is to reconcile this discrepancy between data and the theoretical literature by focusing on the relationship between a country's level of executive constraints and the development of its public sector. There exists an extensive

[4]See Besley, Persson and Reynal-Querol (2014), who find that reform of executive constraints is more likely after the death of a leader who has been in office for a long time. Jones and Olken (2009) reach the similar conclusion that assassinations of undemocratic leaders have triggered positive institutional reforms.

[5]As will be discussed subsequently in the Related Literature section, Besley, Persson and Reynal-Querol (2014) provide evidence that political turnover affects the choice of executive constraints only in weak executive constraint-regimes.

empirical literature documenting the correlation between the democratic attributes of a country and positive indicators of public good provision.[6] These studies have used data on public education enrollment levels, availability of public health care, the quality of infrastructure, or the taxation capacity of the state in order to measure and compare public good provision across countries. Their overall findings indicate that democratic regimes provide better public services to their citizens than autocracies or dictatorships, where the spending preferences of governments lie with targeted political spending as opposed to common interest public goods. Lack of competitive elections or the role of a politically connected elite in non-democratic regimes are among the explanations that the literature has offered into this empirical finding.[7]

The relationship established by these studies between a regime's institutional credentials and the quality of its public good provision constitutes the motivation for the thesis of this chapter. Figure 1.4 documents the correlation between the executive constraints score of a country and its public sector development, measured by the "Government Effectiveness" score from the Worldwide Governance Indicators compiled by the World Bank.[8] The figure clearly indicates that countries with high executive constraint scores are also the countries with high government effectiveness.

[6] See Lott (1999), Lake and Baum (2001), Bueno de Mesquita, Morrow, Siverson and Smith (2003), and Deacon (2009) for empirical evidence on the relationship between public good provision and regime characteristics. See Besley and Persson (2011), Figure 1.9, for evidence of correlation between the executive constraints score from the Polity IV Project and the fiscal capacity of the state, measured by total tax revenues as a share of GDP.

[7] Specifically, see Lake and Baum (2001) for an electoral argument and Bueno de Mesquita, Morrow, Siverson and Smith (2003) for a government capture by the elite argument.

[8] The World Bank governance indicators are collected on a yearly basis and measure governance based on the following six dimensions: Voice and Accountability, Political Stability and Absence of Violence/Terrorism, Government Effectiveness, Regulatory Quality, Rule of Law, Control of Corruption. The data is accessible at www.govindicators.org. A country's score on government effectiveness captures the quality of its public services and the delivery of these services by its civil servants.

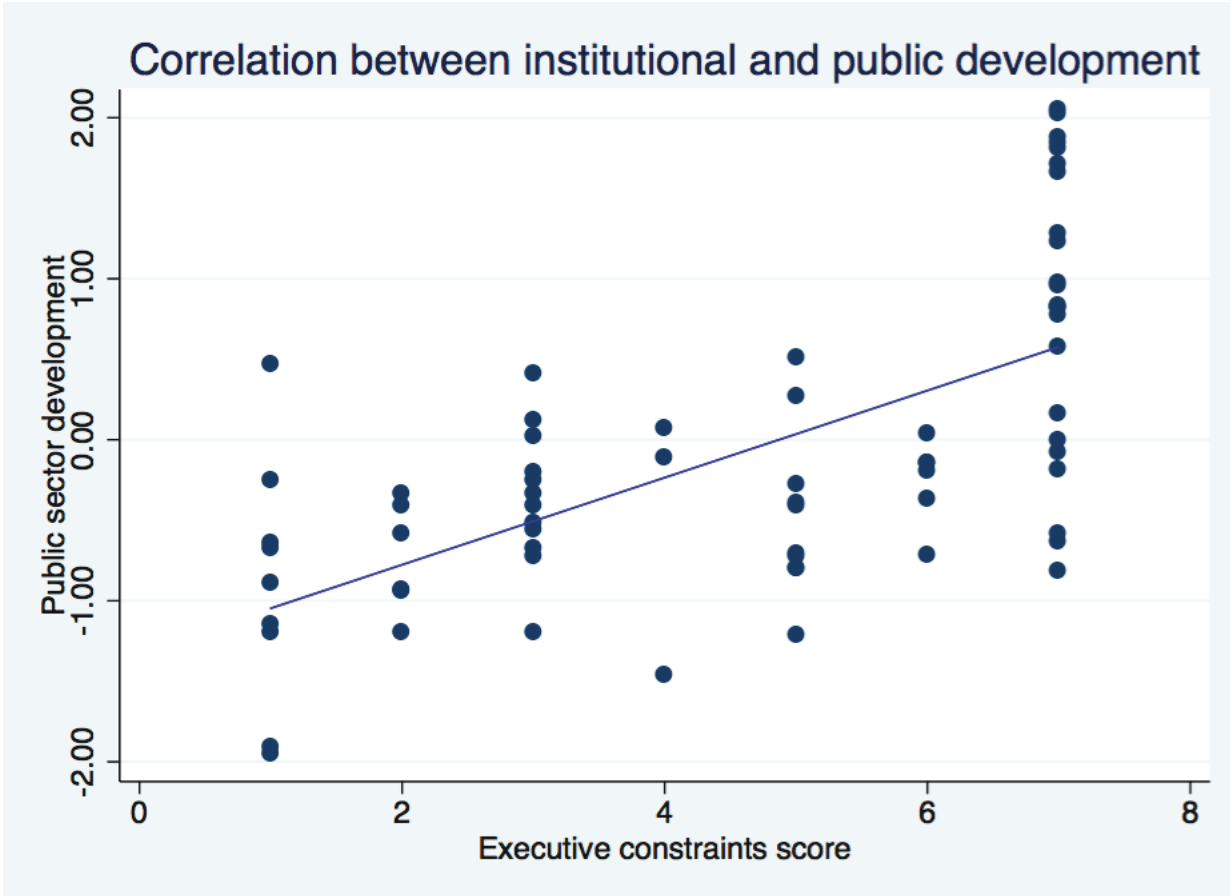

Figure 1.4: Institutional and public sector development are correlated.

This chapter argues that the correlation in Figure 1.4 can potentially explain the differences in the institutional decisions of governments facing similar electoral environments.

To study the determinants of executive constraints in light of Figure 1.4, I focus on the interaction between a government's two types of decisions, institutional and policy, and ask the following questions: How do constitutional limits on executive authority affect the level of public goods and targeted political spending? Which factors contribute to the choice of strong executive constraints and better public good provision? What is the role of political uncertainty in this interaction and how does this role change over time? To address these questions, I build a dynamic model of political competition in which the party in power determines both the policy and the level of executive constraints. I model executive constraints as an endogenous limit on the government's policy choice. The motivation for treating an institution as a constraint on payoff-relevant policies is to emphasize its instrumentality: agents only have intrinsic policy preferences and care about an institution insofar as it enables or inhibits their implementation. Accordingly, an institution as a strategic choice variable is meaningful only in the presence of political uncertainty, because an incumbent would impose constraints on policy-making only as a bulwark against a future government with different preferences.

The model consists of an economy with agents who belong to one of the two groups in the society and who care about their private consumption and a public good. In each period, a representative agent from one of the two groups is exogenously elected to become the new incumbent and unilaterally makes the following decisions: 1) a policy choice that includes the level of investment to a public good that equally benefits all citizens (e.g. environmental or R&D spending) and private transfers of the consumption good to each group (e.g. pork spending); and 2) an institutional choice on the level of executive constraints for the next period. While there exists complete agreement over the public good decision, parties disagree over which group the transfers should be targeted at. Thus, the policy choice incorporates both a common cause and a conflict dimension.

In each period, both a budget and an executive constraint apply to the policy decision. The budget constraint represents the exogenous surplus created in the economy that the incumbent can levy in order to finance his policies. On the other hand, the executive constraint limits the adoption of extreme policy choices. For example, while a budget set imposes no limits on disinvesting the society's entire stock of the public good as long as doing so has a zero price, an executive constraint in this model outlaws such extreme policies. I assume that the executive constraints limit the choice of public goods and private transfers simultaneously. An incumbent picks the levels of investment and pork spending under the executive constraints he inherits from the previous government and chooses those that will constrain the next period's incumbent.

The levels of the public good and the executive constraints constitute the two state variables of the model. I first solve the two-period version of the model, which is sufficient to demonstrate an incumbent's institutional choice trade-off. However, this setting naturally does not allow for a full interaction between the two state variables. Therefore, I extend the model to four periods, which is the minimum number of periods necessary to observe a single feedback loop between the levels of the public good and the executive constraints. The results of this analysis demonstrate an incumbent's complete trade-off in both his policy and institutional choices. I find that for any given distribution of political power between the two parties, the executive constraints get tightened in equilibrium as the public good approaches an optimality benchmark. As the parties alternate in office, the model generates endogenous fluctuations in the level of these constraints. Based on these dynamics, I discuss how public good provision can be improved from its equilibrium levels.

The following trade-off describes the institutional decision in equilibrium: While weak executive constraints permit the incumbent to enact his desired policies in case of re-election, the same applies to his opponent in the opposite scenario. Specifically, the incumbent's motivation for picking looser constraints in order to enjoy levels of investment and private transfers close to his ideals conflicts with the incentive to protect himself from his opponent's future pork spending. The distribution of electoral power determines which of these two effects dominates in equilibrium.

One of the main insights from this analysis is that the above-mentioned role of political uncertainty on an incumbent's institutional decision is not constant. The parties' shared investment interests dominate the conflict over private transfers at too low or too high states of the public good, because there exist large potential mutually beneficial gains to be realized. At these states, the ability to invest (or disinvest) takes precedence over the ability to make political transfers so that both parties discount the role of politics as a result and re-election uncertainty becomes a relatively less important determinant of executive constraints. However, as these gains get exhausted, the parties no longer need to tolerate each other's pork spending for the sake of being able to implement the much-needed investments. Consequently, an incumbent assigns less weight on a future government's freedom to invest and more weight on shielding himself from potential transfers to the other group when choosing the next period's institution.

A similar trade-off applies to the public good decision. Although the parties have a shared incentive to move toward better states of the public good, they also anticipate that this movement would imply a growing role for politics and hence tighter executive constraints in equilibrium, limiting their ability to make transfers to their respective groups should they become the incumbent. As a result, each party holds back on their public good decisions. Moreover, this aversion motive grows as an incumbent's political power increases.

The main theoretical results of this chapter corroborate the empirical evidence by showing that better public good provision is associated with stronger executive constraints that are less sensitive to swings in political power. Despite the parties' shared preferences over it, the public good is still under-provided as the conflict over private transfers takes common cause hostage: Some investment will always be sacrificed to pork politics even at highly sub-optimal levels of the public good.

Confirming the existing results in the literature, I find that the strongest executive constraints arise in societies with the highest political uncertainty. However, this result only holds *for a given level of the public good*. More specifically, the competitiveness of the electoral process can explain the differences in executive constraints only among those countries with similar levels of public sector development. I show that the response of these constraints to shifts in the distribution of political power is more muted in countries that have already exhausted the gains from investments in the public good. These results suggest that other characteristics of a country's public good accumulation process may also interfere with the equilibrium dynamics of its political institutions.

1.2 Related Literature

Focusing on the interaction between the level of executive constraints and the provision of public goods, this chapter contributes to a large literature on the determinants and effects of political institutions. These two complementary branches of the literature have their intellectual foundation in Buchanan's two-stage analysis of public decision-making that consists of a constitutional and a post-constitutional stage.[9] The rest of the section discusses the main studies in the areas to which this chapter contributes.

Determinants of Executive Constraints:

A growing number of studies on the constitutional stage look at institutions that limit executive power. Among them, both the theoretical and the empirical results in Besley, Persson and Reynal-Querol (2014) that link executive constraints to political turnover are the most relevant for the development of the ideas in this chapter. The authors model checks and balances as the share of rents an incumbent must share with the other groups in the society and analyze the policy decision of an incumbent between public goods and private transfers along with an institutional decision. The main theoretical finding of this study is that a high probability of losing office drives the adoption of strong executive constraints. This is supported by empirical evidence, which indicates that the presence of a resilient leader decreases the probability of positive institutional reform by 1 percentage point compared to the incumbency

[9]The *constitutional stage* represents the adoption of rules for policy-making such as voting rules, level of checks and balances, or the system of government. The payoff-relevant policies are then chosen during the *post-constitutional stage* according to these rules. See Buchanan and Tullock (1962) for an in-depth discussion.

of a non-resilient leader. However, the authors find no effect of leadership resilience on the transition from strong to weak executive constraints. While their significance result confirms the importance of leadership change on the determination of executive constraints, the fact that they find no evidence of this in strong regimes offers a compelling motivation for the ideas in this chapter.

An alternative interpretation of checks and balances is offered by Aghion, Alesina and Trebbi (2004), who characterize the optimal size of supermajorities that are constitutionally mandated to pass legislation. Other important papers that endogenize institutions related to accountability and executive constraints include Maskin and Tirole (2004), Ticchi and Vindigni (2010), and Robinson and Torvik (2013).[10] By focusing on the interaction between executive constraints and public sector decisions, this chapter takes a different approach from these studies.

The structure of an institution employed in this chapter is closely related to Lagunoff (2001), who analyzes the determination of legal rules that protect the rights of the minorities as an example of a constraint on majority decision-making. The model features a majority deciding on the set of activities that are permitted to the general citizenry. The main finding indicates that tolerant limits will be imposed even though the majority preferences are not necessarily as tolerant. This arises due to possible interpretation errors on the set of permissible actions and the majority's reluctance to unwillingly impose limits on its own behaviors. His conclusion that societies with higher political uncertainty will exhibit more liberal constitutional rules mirrors this

[10] The choice of voting rule and delegation of authority are other examples of institutions that have led to a number of interesting papers, including Acemoglu and Robinson (2000) and (2001), Lizzeri and Persico (2004), Barbera and Jackson (2004), Jack and Lagunoff (2006), and Lagunoff (2009).

chapter's finding on the increasing tightness of executive constraints.

Acemoglu, Robinson and Torvik (2013) focus on the puzzle that voters have historically voted for politicians who have used their electoral mandate to relax various measures of checks and balances that limited their executive powers. They argue that even though such measures protect the citizenry from the abuse of political power, they also decrease the cost of influencing politicians through bribes. Therefore, citizens face a trade-off between constrained politicians and a corrupt political system. Although different in its focus on voters and corruption, their analysis is relevant to the central questions of this chapter for understanding alternative channels through which executive constraints are influenced.

Implications of Alternative Institutions for Public Good Provision:

By comparing public good provision under different levels of executive constraints, this chapter contributes to the literature on the efficiency implications of institutions. One branch of this literature studies how inefficient policies are generated as the elites use their power to perpetuate the institutions that serve their interests. Prominent examples of studies in this branch include Acemoglu and Robinson (2008), Acemoglu, Egorov and Sonin (2010), and Acemoglu, Ticchi and Vindigni (2011). A more closely related branch compares the performance of public good provision under alternative institutional settings. For example, Lizzeri and Persico (2001) compare the provision of public goods under proportional and majoritarian electoral rules based on the politicians' trade-off between efficiency and targetability of benefits. Battaglini, Nunnari and Palfrey (2012) dynamically study the same question by comparing a legislative decision mechanism with a decentralized one. Bowen, Chen and Eraslan (2014) focus on budgetary institutions of mandatory versus discretionary spending on public goods and find that mandatory public good spending creates more efficient outcomes. They establish their results within a dynamic bargaining model in which the previous year's mandatory spending constitutes today's endogenous status quo. This paper is especially relevant to the present one in terms of its explicit comparative focus on alternative institutional frameworks and modeling of public good decisions within a governmental mechanism. Therefore, along with the previous two papers, it constitutes a building block to the focus of this chapter on the policy implications of executive constraints.

The Implications of Political Power for Public Good Provision:

Without an explicit focus on institutions, Hassler, Storesletten and Zilibotti (2007), Klein, Krusell and Rios-Rull (2008), and Song, Storesletten and Zilibotti (2012) study dynamic policy models with power fluctuations and find public good provision to be inefficient. In contrast to this chapter, the inefficiency result of these studies can be traced to inherent preference differences between political parties over the public good. Azzimonti (2014) also builds a dynamic model of political competition that results in underinvestment, but her model generates this inefficiency not due to fundamental preference differences but due to how a current government can manipulate the level of the public good in order to restrict an opponent's future spending. However, these models all treat the constitutional setting as exogenous and therefore do not allow for a simultaneous analysis of policy and institutional decisions.[11]

[11] This broad literature studies how policies determined by self-interested politicians instead of

Bai and Lagunoff (2011) study a dynamic model of policy-making with endogenous political turnover in which the incumbent faces a trade-off between re-election and implementing desired policies. Theirs is one of the first papers to explicitly model the indirect effects of current policy choices on the identity of future governments in a fully dynamic setting. The resulting dynamics on the co-evolution of governments and policies are relevant for extensions of this chapter to environments that allow for voting and therefore endogenous political turnover.

The underlying framework of this chapter is built upon the ideas in Battaglini and Coate (2007). Within a legislative bargaining model, they analyze the legislators' taxation, private spending, and public good investment decisions. As in the present chapter, the dynamics are created by the previous period's investment decision. They find that for all states of the economy described by a public good that is below an established threshold, investment decisions will be efficient as the public good needs are great. However, as the economy accumulates the public good and the needs decrease, legislators start to underinvest and allocate pork to their districts. The authors focus exclusively on the policy-making aspect of government without allowing the institutional setting to change. This chapter can be seen as an extension of their main idea to an environment where the limits of policy-making designated by the level of executive constraints are also endogenous.

A Re-interpretation of the Modernization Hypothesis:

Finally, the results of this chapter can be interpreted in light of the debate on

benevolent dictators affect the allocation of public resources. Some earlier important papers include Wittman (1989), Persson and Svensson (1989), Alesina and Tabellini (1990), and Besley and Coate (1998). An excellent overview of this literature can also be found in Persson and Tabellini (2001).

the relationship between economic development and democratization. There exists a large literature on the validity of the modernization hypothesis, first put forward by Lipset (1959) who argued that income drives democracy.[12] By emphasizing the feedback loops between strong institutions (proxied by executive constraints) and economic development (measured by the provision of public goods), this chapter provides a basis for a more dynamic interpretation of this hypothesis. I take the view that understanding the determinants and effects of institutions should not necessarily be perceived as separate exercises. Along with others, this study constitutes a beginning in that direction.

The rest of the chapter is organized as follows: Section 1.3 introduces the model and Section 1.4 provides an optimality benchmark by presenting the permanent authority case. Sections in 1.5 characterize the equilibrium under political uncertainty for the two and four period models. The provision of public goods in political equilibrium is compared to the permanent authority benchmark in Section 1.6. Section 1.7 concludes.

1.3 The Model

The model presented in this section is built upon Battaglini and Coate (2007). The economy consists of agents belonging to one of the two groups in the society, A and B. Each agent within a group is homogeneous. Therefore, for simplicity of exposition, I

[12] A more recent treatment on the modernization hypothesis can be found in Przeworski, Alvarez, Cheibub and Limongi (2000). Barro (1999) is one of the most famous papers that reject this hypothesis. Other studies that argue against it include Acemoglu, Johnson and Yared (2008). Che, Lu, Tao and Wang (2012) provide recent supporting evidence.

treat the economy as consisting of two representative agents.

There are two goods in the economy: a private consumption good y and a public good g. Each agent i's preferences are described by a quasi-linear utility function given by

$$u(y_i, g) = y_i + \bar{A}g^{\alpha} \tag{1.1}$$

for $i = A, B$, where $\bar{A}$ is a constant that represents the relative importance of the public good to the private consumption good and $\alpha \in (0, 1)$. I assume that each agent receives his (equal) share $\bar{K}$ of the economy's exogenous surplus of the consumption good. In addition, each agent discounts future utility by the common discount factor β.

In each period t, an election takes place and an incumbent government is realized according to an exogenously fixed probability q_i for $i = A, B$, where $q_A + q_B = 1$. I assume that each agent is purely policy-motivated. The incumbent agent in period t, given by the realization of the state $\kappa_t \in \{A, B\}$ according to q_i, becomes the dictator for that period and unilaterally makes a policy and an institutional decision (without any bargaining with the opposition).[13]

The following subsections describe an incumbent's two types of decisions:

Policy Choice: In each period t, the incumbent κ_t makes the following policy decisions: investment I_t to the stock of g_t; private transfers of the consumption good

[13] Battaglini and Coate (2007) features decision-making within a bargaining framework whereas decisions are taken unilaterally here. In their model, the legislature follows a Baron and Ferejohn (1989) bargaining protocol to make a policy decision that is similar to the one made unilaterally by the incumbent in this chapter. However, this chapter's main interest is in how institutional constraints interact with these policy decisions. Therefore, the present model differs from theirs in its focus on modeling both a given institutional state and the incumbent's institutional decision for the next period.

$y_{A,t}$ and $y_{B,t}$ to agents A and B; and uniform lump-sum taxes $\bar{\tau}_t$ to finance these investment and private transfers.

The model assumes that while there exists complete agreement over the public good decision between the two agents, they disagree over private transfers of the consumption good. The public good represents projects such as military exercises, environmental clean-up, or infrastructure construction that both agents enjoy equally. On the other hand, the transfers are targeted to a specific agent; while both agents pay their share of taxes for a given transfer of good y to agent i, only the type-i agent receives utility from it.

I assume that the public good depreciates each period at the rate δ and its evolution is given by the standard formulation

$$g_{t+1} = (1 - \delta)g_t + I_t. \tag{1.2}$$

Investment is reversible so that there does not exist a non-negativity constraint on its choice. Assuming a production technology given by $g = \frac{y}{p}$ for the public good, the price of a unit of investment can be conveniently represented by p. On the other hand, I assume that disinvestment is free and that its proceeds can be consumed as a private good. I further assume that the cost of providing private transfers is convex. Specifically, the cost of transferring y_i units of the private good to agent i (including the transfer) is given by $x(y_i) = (y_i)^b$, where $b > 1$.

There exists no debt in the model so that the government must balance its budget every period. As a result, the following government budget constraint needs to hold in any given period t:

$$x(y_{A,t}) + x(y_{B,t}) + pI_t \leq \bar{\tau}_t. \tag{1.3}$$

Since an incumbent will never leave tax revenues unspent, 1.3 always holds with equality. A type-i agent's stage utility after the government makes its policy and taxation decisions can be written as

$$\bar{K} + y_i - \frac{\bar{\tau}}{2} + \bar{A}g^{\alpha}. \tag{1.4}$$

Suppressing the constant surplus $\bar{K}$ to reduce clutter and substituting the government budget constraint into 1.4 yield the following stage utility for agent i:

$$y_i - \frac{1}{2}[x(y_i) + x(y_j) + pI] + \bar{A}g^{\alpha}, \tag{1.5}$$

where $j \neq i$. Note that since the government budget constraint always holds with equality and taxation is non-distortionary, the decision on the level of $\bar{\tau}_t$ need not be separately considered.

In addition to the budget constraint that needs to be satisfied each period, an incumbent faces an executive constraint that it inherits from the previous government when choosing a policy. The modeling of this constraint and its choice for the next period are described in the following subsection.

Institutional Choice: Executive constraints are modeled as a limit on the extremeness of an incumbent's policy choice. Let $\Gamma_t \subset \mathbb{R}^3$ denote the institutionally feasible set that represents the level of executive constraints in period t. I assume that all the policies lying on the boundary of the set Γ_t are equidistant from the origin in $\mathbb{R}^3$. In each period, the incumbent chooses the policy vector $(I_t, y_{A,t}, y_{B,t}) \in \Gamma_t$, since the period-$t$ incumbent κ_t inherits the level of executive constraints from his predecessor κ_{t-1}.

Given this structure, let Φ denote the set of all possible institutionally feasible sets. Then, the incumbent's institutional choice is equivalent to defining a set $\Gamma_{t+1} \in \Phi$ by choosing $d(0, z)$ for some $z \in \mathbb{R}^3$, where d is the Euclidean metric and 0 is the origin in $\mathbb{R}^3$. The boundary points $\bar{z} \in \text{argmax}\ d(0, z)\ \forall\ z \in \Gamma_{t+1}$ represent the most extreme policies that are permitted by the executive constraints represented by Γ_{t+1}.

Note that the structure of an institutionally feasible set in this model does not allow an incumbent to impose separate limits on investment and private transfer choices; the executive constraints bind all policy choices simultaneously. The motivation for this structure can be summarized as follows: First, an incumbent should always be allowed to do nothing, i.e. choose a zero level of investment and private transfers to each agent. This implies that the origin should always belong to the set Γ. Second, the model interprets extremeness as the Euclidean distance of a policy from the origin. For example, while a policy that involves very high levels of spending on all three policy dimensions is considered extreme, the same applies to a different policy that allocates all of the economy's tax revenues to one agent's private consumption. As more policies are gradually allowed starting from the origin, a constraint on extremeness means that all the policies lying on the boundary of the institutionally feasible set should be equidistant from the origin.

The underlying rationale for this structure is that an incumbent should not be able to fine-tune his choice of Γ_{t+1}.[14] Choosing a singleton Γ_{t+1} whose only member is the incumbent's ideal policy is one extreme example of such fine-tuning and should be ruled out in order to make the problem interesting. Instead, under the present structure, an incumbent not only ensures he can implement his desired policies if he is re-elected by allowing for more extreme policies in Γ_{t+1}, but also allows his opponent to do the same in the opposite scenario. This trade-off will be important in obtaining non-trivial dynamics from the model.

While it may not be applicable to all policy decisions, the present modeling strategy for an executive constraint is consistent with the design of institutions of interest to this chapter in the real world. Since an institutional reform entails constitutional changes subject to review by a Supreme Court that is mandated to enforce equal treatment clauses, an incumbent is legally barred from designating a feasible set that disproportionately allows his favored policies while outlawing those of his opponent. For example, a party in office has the power to award government contracts to its supporters, but it cannot design a competition agency that only permits the party's supporters to participate in bids for public contracts.

1.4 The Dictatorship Solution

Studying executive constraints as an endogenous limit on policy-making is only non-trivial under political uncertainty; a social planner or a dictator who will remain in office forever would never willingly tie his own hands. However, before analyzing the

[14] An alternative interpretation and modeling of executive constraints that also satisfy this criteria is presented in Appendix A, followed by a discussion on why the present structure is preferred.

model under political competition, characterizing a dictator's optimal investment and private transfers decisions creates a benchmark for evaluating public good provision under political governments.

Without loss of generality, let $q_A = 1$ and $q_B = 0$ so that agent A is the dictator. Let $(I^*(g), y_A^*, y_B^*)$ denote dictator A's ideal policy when the public good is given by g.[15] For a more concise exposition, I assume that the economy's exogenous surplus is sufficient to finance dictator A's ideal policy through lump-sum taxes.

Assumption 1. *For any $g \in \mathbb{R}_+$,*

$$x(y_A^*) + x(y_B^*) + pI^*(g) \leq \bar{K}. \tag{1.6}$$

In the rest of this section, I first solve for dictator A's infinite-horizon problem. Although the political model is only solved for a finite horizon and hence finite horizon dictatorial benchmarks are needed for any comparisons, studying the infinite-horizon solution allows for a better understanding of an agent's investment motives. Second, I solve the finite-horizon problem, which yield a basis for a direct comparison between the dictatorial and the political equilibria. Finally, I discuss why the dictatorial solution is a meaningful benchmark for evaluating the political equilibrium.

Within the infinite-horizon framework, the objective of dictator A is to maximize his dynamic utility.[16] In the absence of an institutional decision, he chooses the level of investment I and the private transfers y_A and y_B in order to solve the following

[15] Note that an institutionally unconstrained dictator has a finite optimal level of private transfers since the cost function $x(\cdot)$ is convex.

[16] Note that agent A is identical to a benevolent social planner who assigns all the weight in the society's aggregate utility to the type-A agent.

program:[17]

$$V^A(g) = \max_{I, y_A, y_B} y_A - x(y_A) - x(y_B) + \bar{A}g^\alpha - pI + \beta V^A(g') \tag{1.7}$$

subject to

$$y_i \geq 0 \text{ for } i = A, B; \tag{1.8}$$

$$I \geq -g; \tag{1.9}$$

$$g' = (1-\delta)g + I. \tag{1.10}$$

Notice that the choices of y_A and y_B are purely static in the dictatorship problem. Moreover, they do not interact with the choice of I as a result of Assumption 1 and the absence of executive constraints. Therefore, by this separability of the transfer and investment decisions, the above problem can be written as the sum of a static and a dynamic component.

First, consider dictator A's static problem:

$$\max_{y_A, y_B} y_A - x(y_A) - x(y_B) \tag{1.11}$$

subject to $y_i \geq 0$ for $i = A, B$. This problem implies that dictator A always chooses $y_B = 0$, because he receives no utility from the private consumption of the type-B agent. Since taxation is non-distortionary, transfers to himself are positive at all states of the public good. Also note that the optimal amount of private transfers

[17] From this point on, I let $x(y_A)$, $x(y_B)$ and pI denote the per-capita costs of these policies in order to limit the amount of notation used.

to the dictator agent is always the same regardless of the identity of the dictator, because both agents enjoy the consumption good linearly and face symmetric costs of providing private transfers. Therefore, the conflict is solely over the target agent.

Second, consider dictator A's investment decision, which constitutes the dynamic component of the problem.[18] Let γ^* denote his equilibrium investment rule such that $\gamma^*(g) = g'$. Then,

$$\gamma^*(g) \in \underset{I \geq -g}{\operatorname{argmax}} \; \bar{A}g^{\alpha} - pI + \beta V^A(g') \tag{1.12}$$

subject to $g' = (1-\delta)g + I$. From the first-order condition for investment, the dictator's Euler equation can be written as follows:[19]

$$\bar{A}\alpha[g']^{\alpha-1} + pD_{g'}\gamma^*(g') = \frac{p}{\beta}. \tag{1.13}$$

The right-hand side of 1.13 is the constant marginal cost of the public good (in tomorrow's dollars). The left-hand side represents the total marginal benefit of the public good: While $\bar{A}\alpha[g']^{\alpha-1}$ is the marginal benefit from its direct consumption, the second term captures the strategic link between periods. Specifically, it is the marginal effect of today's investment decision on future states of the public good and depends on the derivative of an unknown equilibrium rule $\gamma^*(g)$. If the sign of this derivative is positive, today's investments lead to higher future levels of the public good by reinforcing the marginal benefit component. Otherwise, the public good is

[18]The dictatorship problem is fairly standard and previous papers such as Battaglini and Coate (2007) establish the existence of an optimal policy.

[19]Note that the first-order condition yields $D_{g'}V^A(g')D_I g' = \frac{p}{\beta}$, where $D_I g' = 1$ and $D_{g'}V^A(g') = \bar{A}\alpha[g']^{\alpha-1} + \beta D_{g''}V^A(g'')D_{g'}g''$. Taking the first-order condition one period forward yields $\bar{A}\alpha[g']^{\alpha-1} + pD_{g'}g'' = \frac{p}{\beta}$. Substituting the expression for dictator A's equilibrium investment rule γ^*, we obtain 1.13.

inefficiently high and the optimal action is to disinvest.

The solution to 1.12 implies the existence of a threshold level of the public good $\hat{g}$ below which the dictator finds it optimal to invest and above which he disinvests. To see this, suppose g is such that the marginal benefit of investing exceeds the constant marginal cost. At these inefficiently low states, the left-hand side of 1.13 must decrease by investing in g. As g increases, the marginal benefit decreases, eventually hitting the constant marginal cost curve at the point where g equals $\hat{g}$ and the optimal level of investment is zero. Hence, solving the dictator's Euler equation 1.13 for $\hat{g}$ by letting $D_{g'}\gamma^*(g') = 0$ yields

$$\hat{g} = \left(\frac{\beta\bar{A}\alpha}{p}\right)^{\left(\frac{1}{1-\alpha}\right)}. \tag{1.14}$$

For all levels of g above $\hat{g}$, the public good is inefficiently high; the dictator will be disinvesting and consuming the proceeds. Since disinvestment is costless, the dictator always finds it optimal to sufficiently disinvest such that $\gamma^*(g) = (1-\delta)\hat{g}$ for all $g > \hat{g}$.[20]

Figure 1.5 plots the equilibrium investment rule γ^* and shows how the steady-state is determined. Note that the dictator's stage utility and value function $V^A(g)$ are both strictly concave, yielding a unique equilibrium investment rule γ^*. Since the slope of γ^* is always less than 1, there exists a unique steady-state of the public good given by g^*. Moreover, since $\gamma^*(g^*) = g^*$, it must be the case that the function γ^* is increasing at g^*. Therefore, the threshold state $\hat{g}$ must be higher than g^*. Starting

[20]Note that since the public good can be laundered into the consumption good by disinvesting and taxation is non-distortionary, a sufficiently low price for g could make it optimal to invest just to consume these investments in the next period. In order to rule out this possibility, I assume that $p > \beta$ so that it is strictly not optimal to invest beyond $\hat{g}$.

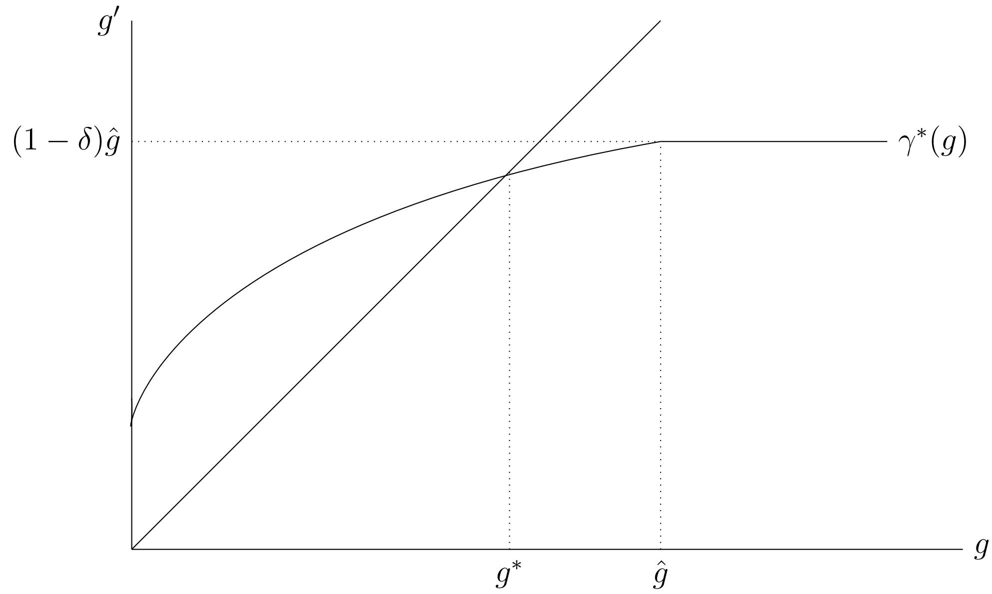

Figure 1.5: Steady-state level of the public good under dictatorship.

from any initial level of g, the public good converges monotonically to this steady-state.

The increasing section of the dictatorship rule γ^* captures both the direct benefit of higher public good consumption and the indirect benefit of having to invest less in the future. Since taxation is non-distortionary, the indirect benefit translates either into tax savings or higher transfers, weakly increasing all agents' private consumption. At these states, the total marginal benefit curve decreases at a decreasing rate but always remains above the constant p. As g increases above $\hat{g}$, the marginal benefit keeps decreasing at a slower rate, reflecting the consumption benefits of disinvestment proceeds.[21]

Since both types of dictators face the same marginal cost and marginal benefit from the public good, they follow the same investment rule in equilibrium. The

[21] Since $p > \beta$, the marginal benefit curve will always lie below p despite the disinvestment proceeds for all $g > \hat{g}$. Note that the different behavior of the marginal benefit curve above and below $\hat{g}$ does not imply a discontinuity in the dictator's value function.

conflict over targeted transfers represents the static component of policy choice and therefore has no effect on the degree of common cause over investments between the two agents.[22] This common investment rule that would be followed under either type of permanent authority provides a benchmark for evaluating public good provision under political equilibrium, because any deviation from $\gamma^*(g)$ decreases the dynamic component of each agent's utility. Therefore, I denote the level of the public good that would be chosen by either type of dictator as the "benchmark state". However, note that this benchmark state does not necessarily correspond to Pareto-optimal levels of the public good, because given the quasi-linear specification of the economy, it may be possible to increase the total utility of the agents by diverting some resources from public good investments to the private consumption of the agent who is not receiving any transfers under dictatorship.

Appendix B characterizes the optimal levels of investment for a dictator when the model consists of four periods. These solutions to the dictatorship problem for each period establish the states that will provide a benchmark for evaluating public good provision in the four-period political model that will be subsequently analyzed.[23] This is because the same argument made in the above paragraph holds when we consider the finite-horizon version of the problem: In the absence of executive constraints that will lead their public good decisions to diverge, both agents maximize the dynamic component of their utilities by choosing the same levels of investment each period. Hence, any level of investment not equal to the solution of the dictatorship problem

[22] This will allow for a more succinct representation of their ideal points in the following section.

[23] This analysis is relegated to the Appendix, because it only serves as a comparative technical benchmark for later analysis and the infinite-horizon solution to the dictator's problem presented above gives a more succinct and intuitive analysis of decision-making under permanent authority.

results in both agents enjoying lower utilities from public good consumption.

1.5 Equilibrium under Political Uncertainty

In the absence of an institutional decision, the optimal policy of an incumbent facing political uncertainty would be equivalent to the dictatorial solution. This is because the decision on private transfers is static and would be separable from the investment decision, and the agents share the same preferences for the public good. Specifically, without a binding budget or executive constraint, incumbent i would choose his static ideal for y_i, make zero transfers to group $j \neq i$, and invest in the public good following the dictatorial rule.

Let $\hat{z}_g^i = (I^*(g), y_A^*, y_B^*)$ denote agent i's ideal policy vector in the absence of executive constraints when the level of the public good is given by g. Since it is never optimal to make positive transfers to the other agent (i.e. $y_{j,t} = 0 \; \forall t$ when $\kappa_t = i$, $j \neq i$), an ideal policy for incumbent i can be described by the vector $\hat{z}_g^i = (I^*(g), y_i^*) \in \mathbb{R}^2$. Figure 1.6 plots these ideal policies for both agents as g changes. In this figure, the x-axis represents investments to the public good and the y-axis represents an agent's private transfers to himself. While the first two quadrants represent a policy space for agent A, the third and the fourth quadrants are for agent B. Although the ideal policies $\hat{z}_g^A$ and $\hat{z}_g^B$ do not lie in the same space, it is convenient to express them as in Figure 1.6 by exploiting the fact that the ideal amount of transfers to the other agent is always zero.

The static nature of the private transfers decision implies that its ideal is constant,

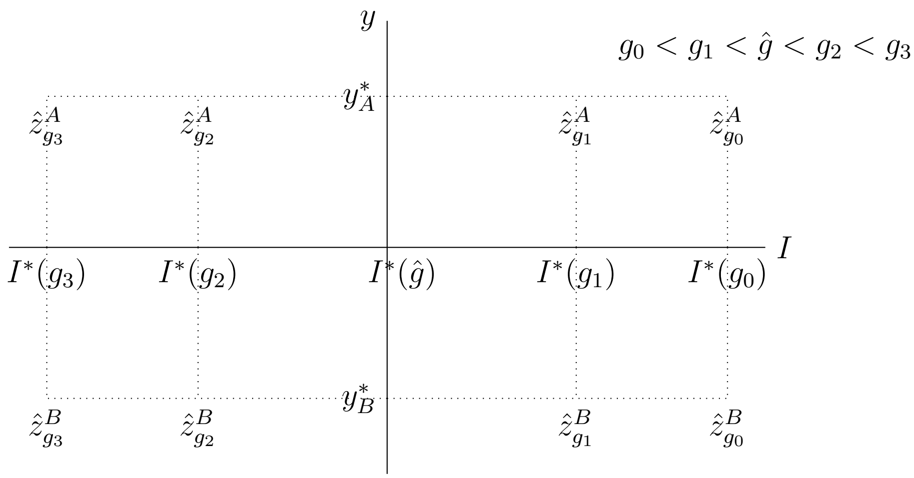

Figure 1.6: The evolution of ideal policies for agents A and B.

denoted by y_i^* for agent $i \in \{A, B\}$. In contrast, the ideal level of investment depends on the state of g and is derived from the dictator's equilibrium investment rule. Starting from a low state such as g_0, investment needs decrease as g increases toward the threshold $\hat{g}$ where the optimal investment equals zero. As g increases above $\hat{g}$ to high levels such as g_2, disinvestment needs increase. As also shown in the figure, the dependence of ideal investments on the stock of the public good implies a corresponding evolution of the parties' ideal policy vectors in $\mathbb{R}^2$ as a function of g.

Recall that the model formalizes executive constraints by the length of the policy vector whose terminal point lies on the boundary of the institutionally feasible set. In $\mathbb{R}^2$, this implies that the Γ_{t+1} choice is equivalent to choosing the radius of a circle, denoted r, with origin at $(0, 0)$. Figure 1.7 illustrates two institutionally feasible sets for a given level of g. While the outer circle permits both agents' ideal policies $\hat{z}_g^A$ and $\hat{z}_g^B$ to be implemented, the smaller circle is more restrictive.

In order to build the intuition for the optimal choice of executive constraints in the following sections, I capture the degree of *polarization* between the two agents

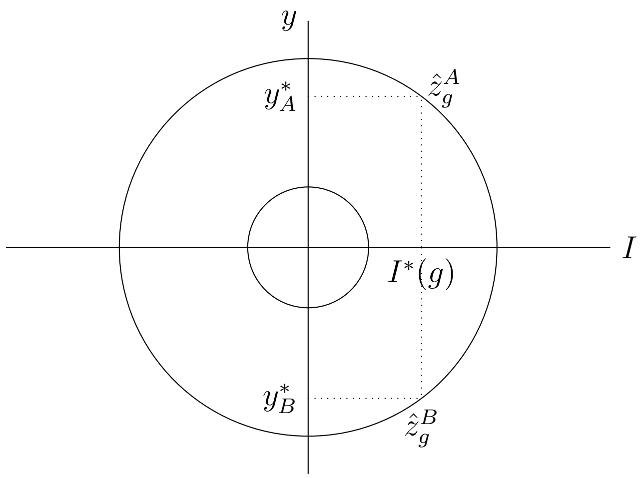

Figure 1.7: Two representations of Γ in $\mathbb{R}^2$.

by the inverse of the difference between the marginal benefit and marginal cost of a unit of investment, given by $\mid \bar{A}\alpha g^{\alpha-1} - p \mid^{-1}$. At very low levels of the public good below $\hat{g}$, the marginal benefit of investing largely exceeds the marginal cost so that the constant disagreement over private transfers translates into a low value of polarization. This is because there exist high potential gains to be realized from investing and common cause is dominant. However, as g increases toward the threshold level $\hat{g}$, the difference $\bar{A}\alpha g^{\alpha-1} - p$ decreases and equals zero at $\hat{g}$. At this point, all the mutually beneficial gains from investing have been exhausted. As g further increases above $\hat{g}$ into the disinvestment range, (the absolute value of) this difference increases again, now reflecting the potential gains from disinvesting and the resulting decrease in polarization. This discussion is summarized in the following remark:

Remark 1. *As the public good moves away from the investment cut-off state $\hat{g}$ in either direction, polarization decreases at an increasing rate.*

The second-order property of the polarization measure is a consequence of the behavior of the marginal benefit curve. It implies that as we move to states of the public good at which investment (or disinvestment) needs are increasingly pressing, common cause starts to dominate the conflict over private transfers at an increasing rate.

To see how the degree of polarization relates to an institutionally feasible set in $\mathbb{R}^2$, let $\bar{r} : \mathbb{R}_+ \to \mathbb{R}_+$ be a function such that the executive constraint defined by $\bar{r}(g)$ is the tightest one that still allows both agents' ideal policies to be implemented when the public good state is g. Figure 1.8 illustrates four different values of the $\bar{r}$ function for some g_0, g_1, g_2 and g_3, where $g_0 < g_1 < \hat{g} < g_2 < g_3$. For example, when the state of g is given by a low level such as g_0 and the corresponding ideal policy for agent i is represented by $\hat{z}^i_{g_0}$ for $i = A, B$, the value $\bar{r}(g_0)$ is given by the radius shown in blue. At this state, the circle with radius $\bar{r}(g_0)$ and origin at $(0, 0)$ allows both agents' ideal policies to be implemented. Likewise, the value of $\bar{r}$ at a state such as g_1 closer to the investment cut-off $\hat{g}$ is represented by the red radius that allows the ideal policy $\hat{z}^i_{g_1}$ for both $i = A, B$.

For any radius $r \in \mathbb{R}_+$, let $\Gamma(r) = \{(a_1, a_2) \in \mathbb{R}^2 \mid (a_1)^2 + (a_2)^2 \leq r^2\}$ denote the circle with origin $(0, 0)$ and radius r. Then, $\bar{r}(g)$ can be interpreted as the shortest radius r for which $(I^*(g), y^*_i) \in \Gamma(r)$ for agent i. The following lemma describes the behavior of the $\bar{r}$ function:[24]

Lemma 1. *For each agent $i \in \{A, B\}$, there exists a function $\bar{r} : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\bar{r}(g) = min\{r \mid (I^*(g), y^*_i) \in \Gamma(r)\}$ is increasing at an increasing rate as g moves away from $\hat{g}$ in either direction.*

[24] All the proofs are in Appendix C.

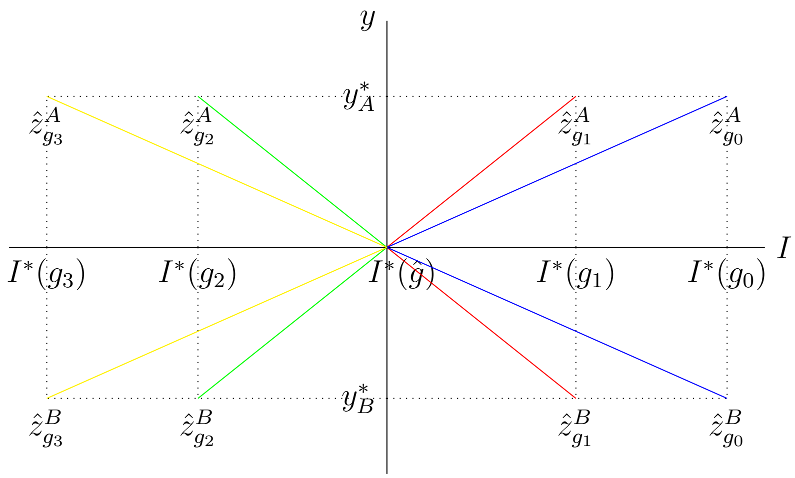

Figure 1.8: Examples of values the function $\bar{r}$ may take.

Note that the function $\bar{r}$ is constructed based only on the agents' ideal policies and does not incorporate equilibrium behavior under political uncertainty. However, because it indicates a tightening of the minimum radius that still permits an agent's ideal policy as g changes, the equilibrium level of executive constraints must be characterized in relation to the values the $\bar{r}$ function takes. To see why $\bar{r}$ is convex as g moves away from $\hat{g}$, notice that it tracks the movement of the marginal benefit of investment and hence of polarization.

In the following section, I begin the equilibrium analysis by formally defining a political equilibrium.

1.5.1 Political Equilibrium

I look for a Subgame Perfect Nash equilibrium (SPNE) to the T-period finite-horizon game described in Section 1.3. At any given period t, the payoff-relevant states of the world are the institutionally feasible set Γ_t (described by the radius r_t from here on) and the public good g_t. An incumbent's equilibrium strategies will be functions of only these two state variables.

Given the executive constraint state r_t and the public good state g_t, a pure policy strategy for incumbent i in period t consists of a pair of private transfer rules $\Upsilon^i_{j,t} : \mathbb{R}^2_+ \to \mathbb{R}_+$ for $j = A, B$ and an investment rule $\gamma^i_t : \mathbb{R}^2_+ \to \mathbb{R}_+$ such that $(\gamma^i_t(r_t, g_t) - (1-\delta)g_t, \Upsilon^i_{j,t}(r_t, g_t)) \in \Gamma(r_t)$. The equilibrium private transfer rules $\Upsilon^i_{j,t}(r_t, g_t)$ for $j = A, B$ yield the amount of private transfers incumbent i allocates to agent j in any given period t and for any level of r_t and g_t. The equilibrium investment rule $\gamma^i_t(r_t, g_t) = g_{t+1}$ yields the level of the public good incumbent i designates for period $t+1$ through his choice of investment in period t. In addition, a pure institutional strategy for incumbent i in period t is an executive constraint rule $\rho^i_t : \mathbb{R}^2_+ \to \mathbb{R}_+$, where $\rho^i_t(r_t, g_t) = r_{t+1}$ yields the radius defining the next period's institutionally feasible set $\Gamma(r_{t+1})$.

Let $\sigma_t \equiv (\sigma^A_t, \sigma^B_t)$ denote a strategy profile for period t, where $\sigma^i_t = (\Upsilon^i_{A,t}, \Upsilon^i_{B,t}, \gamma^i_t, \rho^i_t)$ for $i = A, B$. When the period-t level of executive constraints is given by r_t, the level of the public good is given by g_t, and the agents are following the strategy profile σ_t, let the function $V^i_t(r_t, g_t)$ denote agent i's period-t payoff if he is the period-t incumbent and $W^i_t(r_t, g_t)$ his period-t payoff if he is not in power. Specifically, let

$$V^i_t(r_t, g_t) = \Upsilon^i_{i,t}(r_t, g_t) - x(\Upsilon^i_{i,t}(r_t, g_t)) - x(\Upsilon^i_{j,t}(r_t, g_t)) + \bar{A}g^\alpha_t - p[\gamma^i_t(r_t, g_t) - (1-\delta)g_t)] \tag{1.15}$$

for agent i if $\kappa_t = i$, where $j \neq i$, and let

$$W_t^i(r_t, g_t) = \Upsilon_{i,t}^j(r_t, g_t) - x(\Upsilon_{i,t}^j(r_t, g_t)) - x(\Upsilon_{j,t}^j(r_t, g_t)) + \bar{A}g_t^\alpha - p[\gamma_t^j(r_t, g_t) - (1-\delta)g_t)] \tag{1.16}$$

for agent i if $\kappa_t = j$, where $j \neq i$.

Note that even though Assumption 1 still holds, the investment and the private transfer decisions are no longer separable due to the introduction of executive constraints. Given r_t and g_t in any given period t, the incumbent $\kappa_t = i$ with $T-t$ future periods to live chooses $y_{i,t}$, $y_{j,t}$, I_t, and r_{t+1} in order to solve the following program:[25]

$$\max_{y_{i,t}, y_{j,t}, I_t, r_{t+1}} y_{i,t} - x(y_{i,t}) - x(y_{j,t}) + \bar{A}g_t^\alpha - pI_t \tag{1.17}$$

$$+ \sum_{t+1}^{T} \beta^t [q_i V_{t+1}^i(r_{t+1}, g_{t+1}) + (1-q_i) W_{t+1}^i(r_{t+1}, g_{t+1})]$$

where $j \neq i$, subject to the following constraints:

$$y_{k,t} \geq 0 \text{ for } k = A, B; \tag{1.18}$$

$$I_t \geq -g_t; \tag{1.19}$$

$$(I_t, y_{k,t}) \in \Gamma(r_t) \text{ for } k = A, B; \tag{1.20}$$

$$r_{t+1} \geq 0; \tag{1.21}$$

[25]To reduce clutter, I again suppress the economy's surplus $\bar{K}$ in the incumbent's objective function.

$$g_{t+1} = (1-\delta)g_t + I_t. \tag{1.22}$$

The following is the definition of a political equilibrium to this model:

Definition 1. *A strategy profile* $\sigma_t = (\sigma_t^A, \sigma_t^B)$ *for* $t = 1, ..., T$ *constitutes a Subgame Perfect Nash equilibrium if and only if incumbent* i*'s private transfer rules* $\Upsilon_{A,t}^i(r_t, g_t) = y_{A,t}$ *and* $\Upsilon_{B,t}^i(r_t, g_t) = y_{B,t}$*, investment rule* $\gamma_t^i(r_t, g_t) = g_{t+1}$*, and executive constraint rule* $\rho_t^i(r_t, g_t) = r_{t+1}$ *solve the problem 1.17 subject to the associated constraints 1.18 - 1.22 for all periods* t*, state pairs* $(r_t, g_t) \in \mathbb{R}_+^2$*, and* $i \in \{A, B\}$*.*

In the following section, I solve the model for $T = 2$. The political equilibrium of the two-period model demonstrates an incumbent's full trade-off in his institutional decision. However, this model is not sufficient to observe the full dynamics in an incumbent's policy and institutional choices. Therefore, I solve the model with $T = 4$ in Section 1.5.3, which is the minimum number of periods necessary to have a single feedback loop between the two endogenous state variables of the model.

1.5.2 A Two-Period Model

When the agents live for only two periods, the model can be summarized as follows: The given incumbent at $t = 1$ chooses a policy vector $(I_1, y_{A,1}, y_{B,1})$ for the current period and the level of executive constraints r_2 for the next period, taking g_1 and r_1 as given. At the beginning of period $t = 2$, a new incumbent κ_2 is realized according to the fixed probability q_i for $i = A, B$. Now taking $g_2 = (1-\delta)g_1 + I_1$ and r_2 as given, the incumbent κ_2 chooses the second-period policy vector $(I_2, y_{A,2}, y_{B,2})$. Based on

Definition 1, I solve this two-period model via backward induction for the optimal levels of investment and private transfers in both periods.

First, consider the policy choice of the second period incumbent $\kappa_2 = i \in \{A, B\}$. Since agent i does not receive any utility from the private consumption of agent $j \neq i$, he optimally chooses

$$\Upsilon^i_{j,2}(r_2, g_2) = 0 \tag{1.23}$$

for any state pair $(r_2, g_2) \in \mathbb{R}^2_+$. In contrast, the optimal amount of private transfers incumbent i chooses for himself is given by

$$\Upsilon^i_{i,2}(r_2, g_2) = \min\{r_2, b^{\frac{1}{1-b}}\} \tag{1.24}$$

for any $(r_2, g_2) \in \mathbb{R}^2_+$. In 1.24, $y_{i,2} = b^{\frac{1}{1-b}}$ represents either agent's ideal level of transfers to himself.[26] Whenever $r_2 > b^{\frac{1}{1-b}}$ so that the executive constraints are sufficiently weak, incumbent i is free to transfer this ideal amount to himself. If the opposite is true, this implies that the executive constraint must be binding and the incumbent has to transfer an amount less than his ideal to himself.

Since there are only two periods in this model, the optimal level of investment in the final period equals zero, i.e.

$$\gamma^i_2(r_2, g_2) = (1 - \delta)g_2. \tag{1.25}$$

[26] To see this, maximize the static component of agent i's utility given by $y_i - x(y_i) - x(y_j)$, where $j \neq i$, by choosing y_i. This is agent i's ideal level of private transfers to himself in the absence of a binding executive constraint.

In the first period, the incumbent $\kappa_1 = i \in \{A, B\}$ solves 1.17 for $T = 2$ subject to the associated constraints 1.18 - 1.22, anticipating the optimal second-period choices given in 1.23, 1.24, and 1.25 by future incumbent κ_2. The full solution to the two-period model is summarized in the following proposition.

Proposition 1. *Let $T = 2$ and $\kappa_t = i \in \{A, B\}$ for all t. The Subgame-Perfect Nash equilibrium strategies of incumbent κ_2 is given by 1.23, 1.24, and 1.25. The Subgame-Perfect Nash equilibrium policy strategies of incumbent κ_1 are characterized as follows:*

$$\bar{A}\beta\alpha[\gamma_1^i(r_1, g_1)]^{\alpha-1} - \frac{I_1 - bI_1\left[(r_1)^2 - (I_1)^2\right]^{\frac{b-1}{2}}}{\sqrt{(r_1)^2 - (I_1)^2}} = p, \tag{1.26}$$

which implicitly defines the equilibrium investment strategy $\gamma_1^i(r_1, g_1)$ for any $g_1 \in \mathbb{R}_+$ when the executive constraint r_1 is binding, where $I_1 = \gamma_1^i(r_1, g_1) - (1-\delta)g_1$; and

$$\gamma_1^i(r_1, g_1) = \left(\frac{\beta\bar{A}\alpha}{p}\right)^{\frac{1}{1-\alpha}} \tag{1.27}$$

for any $g_1 \in \mathbb{R}_+$ when r_1 is not binding. Furthermore,

$$\Upsilon_{j,1}^i(r_1, g_1) = 0 \;\forall\; (r_1, g_1) \in \mathbb{R}_+^2 \tag{1.28}$$

where $j \neq i$; and

$$\Upsilon_{i,1}^i(r_1, g_1) = min\{\sqrt{(r_1)^2 - (I_1)^2}, b^{\frac{1}{1-b}}\}. \tag{1.29}$$

The Subgame-Perfect Nash equilibrium institutional strategy of incumbent κ_1 is *characterized by*

$$\rho_1^i(r_1, g_1) = \left(\frac{b}{q_i}\right)^{\frac{1}{1-b}}. \tag{1.30}$$

The optimal choice of executive constraints characterized in 1.30 demonstrates an incumbent's basic institutional trade-off. Before moving on to a discussion of this important trade-off, the following corollary to Proposition 1 summarizes how the optimal choice of r_2 responds to changes in re-election uncertainty.

Corollary 1. *As q_i decreases, the optimal level of $\rho_1^i(r_1, g_1) = r_2$ decreases.*

To see why the executive constraints get tightened as an incumbent's re-election probability decreases, first suppose $q_i = 1$ so that incumbent $\kappa_1 = i$ has no reason to fear his opponent in the future. When there exist only two periods so that the optimal level of I_2 is zero, incumbent i chooses r_2 so as to be able to only implement his ideal level of private transfers given by $y_i^* = b^{\frac{1}{1-b}}$ in the final period. Clearly, any $r_2 \geq y_i^*$ satisfies this.

Now suppose $q_i < 1$ so that there exists a positive probability that incumbent i will have to pay for transfers to the other agent in the second period. Equation 1.30 indicates that as q_i decreases, the first-period incumbent responds to this loss of political power by tightening the executive constraints (since $1 - b < 0$) that will apply to the future incumbent. These optimal levels of r_2 less than y_i^* reflect his incentive to shield himself from the potential undesirable pork spending of his opponent in the second period. The higher the probability that he will not be re-elected, the stronger is this protection motive.

Note that the equilibrium institutional strategy ρ_1^i only reflects incumbent i's private transfer preferences when the agents live for two periods. This is because the optimal level of I_2 always equals zero regardless of the identity of the incumbent.[27] If there existed future periods for today's executive constraint decisions to affect not only the future incumbent's ability to make private transfers to himself but also to invest or to disinvest, the equilibrium strategy ρ_t^i would be expected to reflect an incumbent's public good preferences as well.

In addition to the absence of investment preferences from an incumbent's institutional decision, the fact that $I_2 = 0$ obscures a second trade-off a longer-lived incumbent would face in his public good decision, which will be introduced in the following section. In the current model, there do not exist enough time periods to allow for investment decisions to affect future levels of executive constraints, and vice-versa. In order to demonstrate a full feedback loop between the model's two endogenous state variables, I now turn to the analysis with a longer time-horizon.

1.5.3 Characterizing a Single Feedback Loop

In order to see why we instead need four periods to characterize a single feedback loop between executive constraints and the public good, consider a three-period model: First, since there exist no future periods for g_3 to impact in a three-period economy, the level of I_2 will be identical in equilibrium regardless of the identity of the incumbent. This is because the agents share the same fundamental preferences for the

[27] I assume that $\bar{A}$ is sufficiently large so that disinvesting to consume the public good in the form of consumption good y is not optimal.

public good and the second-period investment is undertaken solely for its consumption benefits in the final period. Second, in addition to identical I_2 choices, either type of incumbent would choose the same level of I_1 in the first period. The intuition for these identical decisions is as follows: The public good state g_2 determined by I_1 affects g_3, but not r_3. This is because the second-period executive constraint choice on the level of r_3 solely reflects private transfer preferences, since the optimal amount of investment in the final period is always zero. With agents deciding identically on the level of g_3, there is no conflict in their I_1 decision. Therefore, an additional period is needed in order to observe differences in induced preferences over the public good stemming from its future impact on the level of executive constraints.

The following proposition presents a full characterization of the four-period model:

Proposition 2. *Let $T = 4$ and $\kappa_t = i$ for $i \in \{A, B\}$ and all t. The Subgame-Perfect Nash equilibrium strategies of each period's incumbent are characterized as follows:*

1. *The Subgame-Perfect Nash equilibrium private transfer strategies of incumbent κ_t are given by*

   – $\Upsilon^i_{j,t}(r_t, g_t) = 0$ *for all t and $(r_t, g_t) \in \mathbb{R}^2_+$, where $j \neq i$.*

   – $\Upsilon^i_{i,t}(r_t, g_t) = min\{\sqrt{(r_t)^2 - (I_t)^2}, b^{\frac{1}{1-b}}\}$ *for all t and $(r_t, g_t) \in \mathbb{R}^2_+$, where I_t is as characterized in item 2 below.*

2. *The Subgame-Perfect Nash equilibrium investment strategies of incumbent κ_t are given by*

   – $\gamma^i_4(r_4, g_4) = (1 - \delta)g_4$ *for any $(r_4, g_4) \in \mathbb{R}^2_+$.*

– *The optimal* I_3 *is as characterized in 1.26 and 1.27. Moreover, the equilibrium rule* γ_3^i *is such that* $\gamma_3^A(r_3, g_3) = \gamma_3^B(r_3, g_3)$ *for all* $(r_3, g_3) \in \mathbb{R}_+^2$.

– *When the executive constraint* r_2 *is binding,* $\gamma_2^i(r_2, g_2)$ *is implicitly defined for any* $g_2 \in \mathbb{R}_+$ *by*

$$\bar{A}\beta\alpha[\gamma_2^i(r_2, g_2)]^{\alpha-1} - \frac{I_2 - bI_2[(r_2)^2 - (I_2)^2]^{\frac{b-1}{2}}}{\sqrt{(r_2)^2 - (I_2)^2}} = p + \beta p \frac{\partial I_3}{\partial g_3}, \tag{1.31}$$

where I_3 *is as given in 1.26. Moreover, the equilibrium rule* γ_2^i *is such that* $\gamma_2^A(r_2, g_2) = \gamma_2^B(r_2, g_2)$ *for all* $(r_2, g_2) \in \mathbb{R}_+^2$.

– *When the executive constraint* r_1 *is binding,* $\gamma_1^i(r_1, g_1)$ *is implicitly defined for any* $g_1 \in \mathbb{R}_+^2$ *by*

$$\bar{A}\beta\alpha[\gamma_1^i(r_1, g_1)]^{\alpha-1} - \frac{I_1 - bI_1\left[(r_1)^2 - (I_1)^2\right]^{\frac{b-1}{2}}}{\sqrt{(r_1)^2 - (I_1)^2}} \tag{1.32}$$

$$+\beta^2 \left[q_i - b\left(\sum_{k=A,B} q_k \Upsilon_{k,3}^i(r_3, g_3)\right)^{b-1}\right]\left[\sum_{k=A,B} q_k \frac{\partial \rho_2^k(r_2, g_2)}{\partial g_2}\right]$$

$$= p + \beta p \frac{\partial I_2}{\partial g_2} + \beta^2 p \frac{\partial I_3}{\partial g_2},$$

where I_2 *is as given in 1.31 and* I_3 *is as given in 1.26.*

3. *The Subgame-Perfect Nash equilibrium institutional strategies of incumbent* κ_t *are given by*

– $\rho_3^i(r_3, g_3) = \left(\frac{b}{q_i}\right)^{\frac{1}{1-b}}$.

– *For any* $(r_2, g_2) \in \mathbb{R}^2_+$, $r_3 = \rho^i_2(r_2, g_2)$ *is implicitly defined by*

$$\left[q_i - b(\Upsilon^i_{i,3}(r_3, g_3))^{b-1}\right] \frac{\partial \Upsilon^i_{i,3}(r_3, g_3)}{\partial r_3} = \mid p - \bar{A}\alpha g_4^{\alpha-1} \mid\mid \frac{\partial I_3}{\partial r_3} \mid, \tag{1.33}$$

where I_3 *is as given in 1.26.*

– *For any* $(r_1, g_1) \in \mathbb{R}^2_+$, $r_2 = \rho^i_1(r_1, g_1)$ *is implicitly defined by*

$$\left[q_i - b(\Upsilon^i_{i,2}(r_2, g_2))^{b-1}\right] \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2} \tag{1.34}$$

$$+\beta q_i \left[q_i - b(\Upsilon^i_{i,3}(\rho^i_2(r_2, g_2), \gamma^i_2(r_2, g_2)))^{b-1}\right] \left[\frac{\partial \Upsilon^i_{i,3}(\rho^i_2(r_2, g_2), \gamma^i_2(r_2, g_2))}{\partial r_2}\right]$$

$$+\beta q_j \left[q_i - b(\Upsilon^i_{i,3}(\rho^j_2(r_2, g_2), \gamma^j_2(r_2, g_2)))^{b-1}\right] \left[\frac{\partial \Upsilon^i_{i,3}(\rho^j_2(r_2, g_2), \gamma^j_2(r_2, g_2))}{\partial r_2}\right]$$

$$= \mid p - \bar{A}\alpha g_3^{\alpha-1} \mid\mid \frac{\partial I_2}{\partial r_2} \mid + \beta \mid p - \bar{A}\alpha g_4^{\alpha-1} \mid\mid \frac{\partial I_3}{\partial r_2} \mid,$$

where I_2 *is as given in 1.31 and* I_3 *is as given in 1.26.*

The equilibrium characterized in the above proposition demonstrates an incumbent's basic trade-offs with regards to his policy and institutional decisions. The first part of Proposition 2 focuses on the static component of policy choice. The fact that agents' private transfer decisions have no future ramifications is the main driver behind the result that the optimal amount of transfers to the other agent is always zero. Therefore, an agent's enjoyment of the consumption good does not play a role on the dynamics of the model. The second and the third parts of Proposition 2 focus on these dynamics in the simplest finite-horizon framework possible.

Part 2 of Proposition 2 characterizes an incumbent's optimal investment decision in each of the four periods when the executive constraint is binding. The left-hand side of equations 1.26 and 1.31 indicate that the marginal benefit of a unit of public good consumption is tempered from its ideal level by the extent to which that period's institution restricts investment. In 1.26, the right-hand side represents the constant marginal cost of investing in period 3. For the period-2 choice, the right-hand side of 1.31 reflects not only this constant marginal cost but also the savings from having to invest less in period 3. Thus, the period-2 investment choice includes the dynamic linkage between periods that we would observe in a standard capital accumulation problem.

The more important observation from 1.26 and 1.31 for the purpose of this chapter is the non agent-specificity of the I_2 and I_3 decisions. The intuition for why either type agent would make the same I_3 decision when in power is straight-forward: g_4 is payoff-relevant only for its consumption benefits. Since the agents have identical preferences over public good consumption, they would invest the same amount in the period leading to the final one. To see why the optimal I_2 decision also does not depend on the type of incumbent, note that while g_3 affects the level of g_4, it does not affect r_4. This can be observed from the fact that $\rho_3^i(r_3, g_3)$ depends only on the parameters b and q_i. Specifically, the r_4 decision is motivated solely by an agent's private transfer preferences in the final period (since $I_4 = 0$). Therefore, when deciding on the level of g_3, the period-2 incumbent only considers its effect on g_4. Because g_4 only yields consumption benefits, the resulting I_2 decision is conflict-free.

Conflict in the agents' investment decisions starts to exhibit itself in equation 1.32, which characterizes the optimal I_1. The presence of the agent-specific re-election parameter q_i in this equation results in the dependence of I_1 on the identity of the agent in power. A closer inspection of 1.32 suggests that this dependency arises due to the effect of I_1 on the future private transfers that either agent could potentially decide on if they come to power. Since this effect works through future states of executive constraints, I will first discuss their equilibrium behavior before offering a more detailed explanation of why agents make different I_1 decisions.

Consider the equilibrium institutional strategies characterized in Part 3 of Proposition 2. The intuition for the optimal r_4 is the same as in the two-period problem summarized in Corollary 1. In contrast to this decision that only reflects an incumbent's private transfer preferences, equations 1.33 and 1.34 indicate that the optimal r_3 and r_2 decisions reflect an incumbent's preferences for both policy components. Specifically, incumbents choose r_3 and r_2 so that the net marginal benefit from the additional private transfers enabled by a unit increase in r exactly equals the (negative) net marginal benefit from the additional investments enabled by the same unit increase. To see this more clearly, re-arrange 1.33 and focus on the range $g < \hat{g}$ where there exist positive investments to get

$$q_i \frac{\partial \Upsilon^i_{i,3}(r_3, g_3)}{\partial r_3} + \bar{A}\alpha g_4^{\alpha-1} \frac{\partial I_3}{\partial r_3} = p \frac{\partial I_3}{\partial r_3} + b(\Upsilon^i_{i,3}(r_3, g_3))^{b-1} \frac{\partial \Upsilon^i_{i,3}(r_3, g_3)}{\partial r_3}. \tag{1.35}$$

The left-hand side of 1.35 represents the total marginal benefit and the right-hand side represents the total marginal cost of a unit increase in r_3. Specifically, if incumbent κ_2 chooses a marginally higher r_3, thereby relaxing the executive constraints for period 3, he receives the following benefits: First, an expected benefit of q_i from each extra unit of private transfers enabled by the marginally higher r_3, and second, the marginal benefit from public good consumption, measured by $\bar{A}\alpha g_4^{\alpha-1}$, for each extra unit of investment. When choosing r_3, the incumbent weighs these benefits against the costs of such a relaxation, which consist of the constant marginal cost of investing and the marginal cost of private transfers. The optimal r_3 decision occurs at where the marginal benefits equal the marginal costs. Equation 1.34 for the optimal choice of r_2 indicates a similar intuition, extended to an additional future period. Notice that since private transfer preferences always play a role on the institutional decision, the agent-specific parameter q_i appears on all the equilibrium values of $\rho_t^i(r_t, g_t)$. Therefore, as long as agents face different re-election probabilities, their optimal executive constraint decisions will be different.[28]

The following proposition describes how incumbents respond to changes in the distribution of political power when making their institutional choices:

Proposition 3. *In the Subgame Perfect Nash equilibrium characterized in Proposition 2, the value of $\rho_t^i(r_t, g_t)$ is increasing in q_i for all t and $i = A, B$.*

Proposition 3 affirms the main results in the existing literature by asserting that the executive constraints get tightened as an incumbent's re-election prospects deteriorate and weakened as they improve. As an incumbent becomes more confident of his ability to determine policy in the next period, he relaxes the rules that will constrain his policy choices. Specifically, he exploits his political advantage in order

[28]Even though r_2 and r_3 decisions reflect an incumbent's preferences for both types of policies, note that the agents make identical investment decisions in $t = 2$ and $t = 3$. Therefore, private transfer preferences are the only reason why agents make different executive constraint decisions in these periods. In order to observe differences in the r decision attributable to differences in investment preferences, we would need to extend the model to $T = 5$.

to freely transfer more resources to himself to the potential detriment of his opponent. However, note that Proposition 3 holds only for given states of the public good. In other words, the predicted response of executive constraints to changes in re-election uncertainty is for a constant level of public good development. It is therefore uninstructive to compare this response across economies who are at different stages of development.

The following proposition describes the behavior of executive constraints as the public good varies:

Proposition 4. *In the Subgame Perfect Nash equilibrium characterized in Proposition 2, the value of $\rho_t^i(r_t, g_t)$ decreases as polarization increases for all t, $i \in \{A, B\}$, and any given level of q_i. Moreover, the difference between $\bar{r}(g_t)$ and $\rho_t^i(r_t, g_t)$ is positive for all $(r_t, g_t) \in \mathbb{R}_+^2$.*

Proposition 4 states that for any given level of re-election uncertainty, the executive constraints get tightened as the level of g approaches the investment cut-off state $\hat{g}$ from either direction. At too low or too high states of g, the common investment needs of the society are sufficiently pressing that incumbents have an incentive to keep the executive constraints loose regardless of the distribution of political power. These are the states of the public good at which polarization between the agents is low. As the economy moves away from these states by investing or disinvesting, the constant disagreement over private transfers translates into higher measures of polarization, resulting in tighter executive constraints.

Proposition 4 offers an important qualification to the main results established in the literature and re-iterated in Proposition 3: While the executive constraints do respond to re-election uncertainty, the extent of this response varies based on the economy's level of public good development. Comparing two countries that are characterized by the same distribution of political power but different states of the public good, Propositions 3 and 4 together suggest that the executive constraints in the less developed country would respond more to a given change in re-election uncertainty than those in the developed country. Recall that since an executive constraint simultaneously restricts both policy dimensions, its optimal level will be co-determined by the incumbent's investment and private transfer preferences. This chapter predicts that private transfer preferences play a relatively more important role on the determination of executive constraints in the developed country because of the disappearance of common cause between its agents.

Having described the behavior of the executive constraints with respect to re-election uncertainty and public good development, I return to the discussion of an incumbent's investment decision. Equation 1.32 demonstrates an incumbent's public good trade-off that emanates from having executive constraints as a strategic choice variable and results in different I_1 decisions despite shared public good consumption preferences. In the absence of an institutional decision, each type of incumbent would invest by the same amounts, because the only dynamic effect of investing (disinvesting) would be higher (lower) future levels of the public good. In a model in which the public good affects agents' utilities only through its consumption benefits, there would be no reason to expect different investment behavior. However, in the present model, the public good also determines the level of polarization between agents, thereby affecting their choice of executive constraints. Since executive constraints imply restrictions on the private transfers an agent can carry out (over which there is conflict), we observe different induced preferences over the public good. The following proposition focuses on the source of this difference.

Proposition 5. *In the Subgame-Perfect Nash equilibrium characterized in Proposition 2, the difference between $\gamma_1^A(r_1, g_1)$ and $\gamma_1^B(r_1, g_1)$ is increasing in $\mid q_A - q_B \mid$.*

Proposition 5 formalizes the idea that an incumbent's investment decision cannot be motivated solely by a shared interest in public good consumption when executive constraints are subject to strategic choice. Specifically, even though an incumbent still has an incentive to accelerate the accumulation (or the decumulation) of the public good towards the investment cut-off state $\hat{g}$, he is also aware of the tighter executive constraints such states of g would imply, impairing his ability to make transfers to himself should he become the incumbent. This tightening effect, shown in Proposition 4, introduces an aversion motive to the investment decision. As an incumbent becomes more confident that he will be the agent determining private transfers in the next period, this aversion motive that slows the movement of g towards $\hat{g}$ grows stronger.

Propositions 3, 4, and 5 together demonstrate an incumbent's full trade-off based on the endogenous feedback-loop between the states of g and r. Because the agents equally enjoy the consumption of the public good and private transfers is a static decision, the only source of dynamic asymmetry in the model is political power. The extent of this asymmetry leads to varying degrees of divergence between the agents' induced preferences over executive constraints. The next section focuses on the implications of this divergence for public good accumulation by comparing it to the dictatorial benchmark established and discussed in Section 1.4.

1.5.4 An Evaluation of Political Equilibrium

The results established in the previous section indicate that identical preferences for public good consumption does not translate into identical investment decisions. To reiterate, the state of the public good affects the optimal level of executive constraints by determining the degree of polarization between the agents. In turn, incumbents choose future states of the public good under these constraints, mindful of their decision's implication for future degrees of polarization and hence for executive constraints. Therefore, with an institutional structure that restricts the extremeness of policies instead of placing separate bounds on the level of each policy dimension, the investment decision can no longer be uncoupled in equilibrium from the incumbent's preference for making transfers to himself.

With exogenously given executive constraints, the investment decision of each agent would be politics-free and hence identical, driven entirely by their shared preferences over the public good. However, in a dynamic setting with endogenous executive constraints, this is no longer the case. Based on this discussion, the following proposition compares the provision of the public good in political equilibrium with the dictatorial benchmark:

Proposition 6. *Compared to the dictatorial benchmark, the provision of public goods in political equilibrium is sub-optimal.*

There are two main sources to the result in Proposition 6: First, the disagreement over private transfers between the agents lead an incumbent to restrict future investments or disinvestments despite shared preferences over public good consumption. Second, investment is further held back by incumbents who fear the increase in polarization and the consequently tighter constraints better states of the public good would bring. The underlying reason behind both can be traced to universal taxation for financing agent-specific consumption. Accordingly, the political equilibrium would be equivalent to the dictator's equilibrium in the absence of distributive politics. Similarly, the dictatorial benchmark as defined in Section 1.4 that maximizes both agents' utilities from public good consumption would be restored if each agent could credibly commit to making sub-optimal transfers to himself and maintaining weak executive constraints. However, the absence of a commitment technology rules out this possibility.

Based on Proposition 6, the question arises as to which parameters of the model amplify this result. Since executive constraints are its main driver, parameter shifts that influence the balance an incumbent strikes between his private transfer and investment preferences when choosing the optimal level of executive constraints can be expected to play a role. Consequently, the difference between a dictator and incumbent's investment rules increases with the uncertainty of elections and decreases with the common-cause parameter $\bar{A}$ and the cost parameter b.

To see why incumbents facing less electoral uncertainty would invest similarly to a dictator, note that an incumbent's problem approaches that of a dictator as his political power becomes absolute. The weakening of his protection motive as his political power increases implies that he optimally chooses looser executive constraints. In the limit as q_i approaches 1, incumbent i picks the level of executive constraints so as to be able to implement his ideal policy vector, which includes dictatorial investment levels. The public good provision would be equivalent to the dictatorial solution, but this comes at the expense of the type-j agent paying for unrestricted transfers to agent i. Therefore, the equilibrium would still not be Pareto efficient.

Similar to political uncertainty, the parameters b and $\bar{A}$ affect the political equilibrium by changing the terms of an incumbent's institutional trade-off. Specifically, an increase in b decreases an agent's static optimum for private transfers to himself, weakening the incentive to restrict each other's pork spending. As a result, executive constraints would be kept weaker at all states of g, resulting in less restrictions on investments. Likewise, an increase in $\bar{A}$, for instance due to a natural disaster, increases either agent's optimal investment at any given state of g. This results in a higher weight on investment preferences in the incumbent's institutional decision, leading to weaker executive constraints.

1.6 Concluding Remarks

This chapter analyzed the dynamics between political institutions that limit executive decision-making and the provision of public goods. With a dynamic model of political competition between two policy-motivated agents, I characterized the trade-off between productive investment and pork spending when both the policies and the institutions are endogenous.

The states of the public good and the executive constraints constitute the dynamic linkages between periods. An incumbent's institutional decision is shaped by his opposing incentives to push the public good to its dictatorial benchmark level while restricting his opponent's potential private transfers in the future. The degree of polarization between the agents and the exogenous distribution of political power determine which of these effects dominates in equilibrium. The feedback loops between the two state variables are created as forward-looking incumbents anticipate the increase in polarization better states of the public good would imply. As higher polarization leads to tighter executive constraints that limit each agent's future ability to make private transfers to himself, investment is held back by an aversion motive. As a result, public good provision is sub-optimal in equilibrium compared to the dictatorial benchmark even though the agents share the same preferences over its consumption.

The dynamics of the model offer an insight into why countries with higher levels of public sector development and strong political institutions retain their institutions: If we consider two societies with similar distributions of political power but different stages of public good development, the executive constraints of the more developed country will be oscillating within a narrower band than those of the less developed country for a given shift in political power. In other words, the executive constraints in less developed countries are more sensitive to the electoral environment. The main driver behind this result is the role polarization plays on an incumbent's institutional decision.

The theoretical results of this chapter lead to some interesting positive implications. First, I find that better states of the public good are associated with tighter executive constraints, corroborating the existing evidence. Accordingly, exogenous factors that derail or boost a country's public good accumulation process will affect its institutional development. The second implication I consider concerns an old debate on the effect of competitive politics on economic development. Specifically, my results imply that while competitive elections hurt public good accumulation during the early stages of development, they lead to a more balanced political spending regime. In addition, I show that a distribution of political power that does not award an overwhelming advantage to one agent is necessary for enacting tight constraints on executive decision-making.

For tractability, I make a number of modeling choices that could be modified. For example, I exogenously fix re-election probabilities so that the incumbents only consider the effect of their decisions on future policies. Explicitly introducing voter preferences into this framework would certainly yield richer dynamics, but would come at the expense of higher complication. Another possible extension involves endowing the agents with institutional preferences, thereby diverging from the pure instrumentality assumption on executive constraints. It is also important to note that this chapter focuses on one among numerous possible structures for an institutionally feasible set. Although I believe that the equidistant-from-the-origin approach is a reasonable proxy for considering institutions that limit extremeness of policy choices, different structures could be more appropriate for settings in which incumbents have the ability to impose separate limits on policies.

Finally, this chapter is silent on the role of social classes in bringing about institutional change. Explicitly introducing groups such as elites, a middle class, or the military will potentially yield rich results on their interaction. This branch of the literature offers various interesting areas for future research.

Chapter 2

A Model of Democratic Capital Accumulation

2.1 Introduction

This chapter continues the focus of the dissertation on political institutions related to curbing the decision-making powers of the government's executive branch. In Chapter 1, I built a model of political competition in which the incumbent governments strategically determine the future level of executive constraints in order to explain the following empirical observation: While there exist countries with strong executive constraints that have managed to retain them over time, the institutions of others exhibit fluctuations between strong and weak periods.[1] The first two sections in Chapter 1 discussed the existing literature's theoretical finding of a link between

[1] The measurement of executive constraints is based on the same Polity IV Project data as in Chapter 1.

strong constraints on executive decision-making and the uncertainty of election outcomes. However, the empirical evidence in Besley, Persson and Reynal-Querol (2014) confirms that this relationship only holds in weak executive constraint-regimes.

Based on these observations, which were more extensively discussed in Chapter 1, the question arises as to which factors besides the distribution of political power explain the divergence between countries' institutional decisions. More generally, why do strong institutions of checks and balances persist under certain regimes but not in others, even when these regimes share electoral characteristics? The previous chapter offered to fill the gap between the empirical and theoretical results by focusing on a country's public sector development. As an alternative, this chapter considers the explicit costs of institutional reform as a possible explanation into this quagmire.

In order to study the effect of institutional reform costs on the determination of executive constraints, this chapter builds a model of political competition in which an incumbent can make reversible investments into the future incumbent's ability to reform the level of executive constraints. The model features two political parties, one of which becomes the incumbent in each period. As before, I exogenously fix the re-election probabilities in order to focus exclusively on an incumbent's trade-off in reforming the executive constraints by abstracting away from the potential electoral consequences of such reforms. In each period, the incumbent decides on a policy, the level of executive constraints, and the amount of investment into the economy's stock of "democratic capital", which constitutes a measure of the difficulty of institutional reform. An incumbent chooses a policy under the executive constraints he inherits from the previous government and chooses those that will limit the future incumbent's policy choice. If the incumbent chooses to change the existing level of executive constraints for the next period, he incurs a cost that is equal to the stock of democratic capital in the economy. For simplicity, I assume that this out-of-pocket cost does not vary with the extent of institutional reform. In addition, the incumbent can influence the future stocks of democratic capital by investing or disinvesting in it. Hence, executive constraints and the democratic capital stock constitute the two endogenous state variables of the model.

The analysis starts with a benchmark model that does not incorporate democratic capital either as an exogenous stock or as a strategic choice variable. The purpose is to replicate the results of the previous literature in order to demonstrate its shortcomings. Specifically, I build a simple two-period model in which today's incumbent faces an exogenous re-election probability and determines the executive constraints under this uncertainty that will limit tomorrow's policy choice. In this environment, the incumbent has an incentive to decide on weaker constraints if he is likely to be re-elected so that he can more easily implement his desired policies. On the other hand, if his opponent is likely to be the future incumbent, he optimally chooses to tighten the executive constraints. This is due to the same protection motive from the opponent's undesirable policy choice identified in the previous chapter.

As mentioned above, this theoretical result fails to hold empirically as we observe incumbents with similar re-election prospects behaving differently under their respective regimes. This chapter introduces differences in the societies' stocks of democratic capital as an explanation for this discrepancy between theory and empirics. The main finding indicates that greater electoral uncertainty lead to tighter executive constraints and higher stocks of democratic capital. These results shed light on our empirical observations: The strong executive constraints of those countries that have persistently had competitive elections will withstand one-time electoral advantages by a political party, because both incumbents will have contributed to the accumulation of the society's stock of democratic capital in the past. I also emphasize the importance of having political parties with polarized policy preferences for democratic capital accumulation to take place. Otherwise, the protection motive that propels an incumbent to invest in the difficulty of institutional reform would disappear. On the other hand, when re-election probabilities favor one party, the democratic capital stock of the economy fluctuates as it is only the underdog party who invests whenever he gets to be in office.

While it is convenient to model the difficulty of implementing institutional reforms as an out-of-pocket expense that needs to be paid by the incumbent at the time of the reform, this modeling choice is intended to represent broader aspects of institutional processes that are more challenging to quantify. The goal of this chapter is to capture through the democratic capital variable the institutional provisions that make it more difficult for governments to tinker with the constraints that bind them. For example, we may think of the power enjoyed by the opposition in a political bargaining framework as representing the difficulty of institutional reform. The stronger the opposition is (as a result of quorum or filibuster rules, for example), the more difficult it is for the incumbent to change the level of executive constraints to its advantage. Another example is a strong judiciary with the right to overrule institutional decisions. For both examples, incumbent governments rarely view the costs of institutional reform as untouchable. Instead, they can take actions that would increase or decrease these costs. Understanding the factors that lead an incumbent to choose to increase them is the subject of this chapter.

2.2 Related Literature

This chapter contributes to a large literature on the determinants of political institutions. However, in contrast to the first chapter, it is silent on its effects.[2] Since the first two chapters of this dissertation analyze the same political institution of executive constraints and are motivated by the same empirical observations, all the studies discussed in Section 1.2 are closely related.

By providing its main motivation, Besley, Persson and Reynal-Querol (2014) remains the most relevant study to this chapter. To re-iterate, the authors establish a theoretical link between political turnover and the strength of executive constraints. They identify an incumbent's trade-off between weaker constraints that enable him to implement his desired policies and stronger constraints that offer protection from his opponent in case he loses office.[3] However, as extensively discussed in Chapter 1 and the introduction to this chapter, this result contradicts the observed stability of strong political institutions in democratic regimes whose incumbents are facing a one-time electoral advantage.

An important study that was not previously discussed in Chapter 1 is Persson

[2]The first chapter of the dissertation had a dynamic policy component and yielded results on the long-term effects of executive constraints on a public good.

[3]This result is replicated in the benchmark model in Section 2.5.

and Tabellini (2009), which is closely related to this chapter's focus on the explicit costs of institutional reform. The authors are one of the first to explicitly incorporate a democratic capital variable, measured by the years of democratic rule in a country, into a formal analysis in order to explain differences in institutional outcomes. Within an overlapping-generations model in which the returns to investment depend on the probability of regime change, they find that higher endowments of democratic capital spur a country on a virtuous cycle of political and economic development. This result is due to higher levels of democratic capital making regime transitions into autocracies less likely, thereby increasing the expected returns on investment and hence growth. While this chapter interprets the democratic capital stock of a country in a different context related to the cost of reforming executive constraints, its results that predict positive effects of democratic capital accumulation on political development mirror theirs.

Among the growing literature that studies the determinants of political institutions, and specifically those that look at executive constraints, a number of them can be interpreted as implicitly studying the costs of institutional reform. For example, Aghion, Alesina and Trebbi (2004) interpret the checks and balances that limit executive power through the size of the supermajorities needed to block legislation. In that sense, their supermajority requirement variable is analogous to the cost variable in this chapter. However, this requirement is a static decision and therefore is silent on the long-term determinants of such provisions. Although it does not study institutions per se, the capital accumulation in Azzimonti (2014) that manipulates the spending decisions of future governments can be interpreted in an analogous light to the democratic capital accumulation that takes place here. Other papers open to such possible interpretations include Maskin and Tirole (2004), Ticchi and Vindigni (2010), Robinson and Torvik (2013), and Acemoglu, Robinson and Torvik (2013). However, to my knowledge, this chapter is the first to simultaneously study institutional reform and the difficulty of implementing such reform.

The rest of the chapter is organized as follows: The next section introduces the model. Section 2.4 solves for the dictatorial solution in order to better understand the incentives political parties face under political uncertainty. Section 2.5 focuses on a simple benchmark model with no democratic capital variable in order to demonstrate how this model is related to the existing literature and how I aim to contribute to it. Section 2.6 defines a political equilibrium and characterizes it. Section 2.7 concludes.

2.3 The Model

The economy consists of two agents, A and B. We can think of these two agents as representing two different groups within the society. In each period t, one of the agents $i \in \{A, B\}$ is exogenously elected with probability q_i to serve as the incumbent, where $q_A + q_B = 1$. The agents are purely policy-motivated so that they seek office only for the purpose of being able to implement their desired policies.

Let $p_t \in \mathbb{R}$ denote a policy in period t and let $\hat{p}_i \in \mathbb{R}$ denote the ideal policy of agent i.[4] Agent i's preferences are represented by the Euclidean distance function

$$u_i(p_t) = -d(\hat{p}_i, p_t), \tag{2.1}$$

where d is the Euclidean metric. Each agent discounts future utility by the common discount factor β. I assume that the incumbent acts unilaterally during his tenure in office to make a policy choice, an institutional choice, and an investment choice into the democratic capital stock of the society. Each of these decisions are respectively described in the following subsections:

Policy Choice: In each period t, the incumbent chooses a policy $p_t \in \mathbb{R}$. The policy choice is static in the sense that all of its benefits are consumed in the current period. Equation 2.1 implies that an incumbent's stage utility is maximized when the policy is set at his ideal. However, this policy choice is constrained by the institutional framework. Specifically, each incumbent maximizes his utility by choosing a policy p_t subject to the executive constraints that he inherits from the previous incumbent. The structure of an executive constraint is discussed in the following subsection.

Institutional Choice: As in Chapter 1 of the dissertation, I model executive constraints as a limit on the extremeness of an incumbent's policy choice. Let $\Gamma_t \subset \mathbb{R}$ denote the institutionally feasible set that represents the level of executive constraints

[4]For example, a policy may represent an income tax rate or the level of spending on a welfare program.

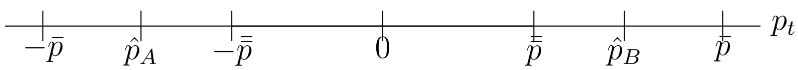

Figure 2.1: Two examples of Γ_t when ideal policies are given by $\hat{p}_A$ and $\hat{p}_B$.

in period t. I assume that the policies lying on the boundary of this set are equidistant from the point zero.[5] Therefore, an institutionally feasible set is an interval with zero as its median point. Since the period-t incumbent inherits the constraints represented by Γ_t from the previous incumbent, he chooses $p_t \in \Gamma_t$. Furthermore, he makes an institutional decision for tomorrow by designating Γ_{t+1}. Figure 2.1 demonstrates two examples of institutionally feasible sets. While the interval $[-\bar{p}, \bar{p}]$ represents a Γ_t that permits the implementation of both agents' ideal policies, the smaller interval $[-\bar{\bar{p}}, \bar{\bar{p}}]$ represents more restrictive executive constraints.

Given this structure, let Φ denote the set of all possible institutionally feasible sets. Then, choosing $\Gamma_{t+1} \in \Phi$ is equivalent to choosing $d(0, z)$ for some $z \in \mathbb{R}$, where d is the Euclidean metric in $\mathbb{R}$. The policies $\bar{z} \in \text{argmax } d(0, z)\ \forall\ z \in \Gamma_{t+1}$ represent the most extreme policies permitted under the institutional choice Γ_{t+1}. Furthermore, this implies that the Γ_{t+1} choice can be thought of as choosing the length of a line-segment, denoted ℓ, whose midpoint is zero. For any $\ell \in \mathbb{R}$, I let $\Gamma(\ell)$ denote the line segment with length ℓ and midpoint zero.

The motivation for imposing this structure on an institutionally feasible set is discussed in the previous chapter. To reiterate, the most important property of Γ_t as defined above is to rule out "fine-tuning" by either incumbent. Specifically, this structure ensures that an incumbent cannot designate an institutionally feasible set

[5]Equidistance from any fixed point yields identical results.

that gives himself an overwhelming policy advantage in the next period. For example, under the present Γ_t structure, incumbent i cannot choose tomorrow's set of permissible policies so that it consists solely of his own ideal policy, i.e. choose $\Gamma_{t+1} = \{\hat{p}_i\}$. Doing so would leave tomorrow's incumbent with no choice but to implement that policy and yield only trivial results. Overall, an incumbent must face the following trade-off in his institutional decision: To the extent that the incumbent weakens the level of executive constraints by designating a greater interval of permissible policies for tomorrow, he not only allows himself to move closer to his ideal policy should he get re-elected, but also allows his opponent to do the same in the opposite scenario. This is dangerous from his point of view, because it would imply moving *away* from his ideal policy as long as agents have polarized policy preferences.

Investment Choice to the Stock of Democratic Capital: In addition to choosing a policy p_t from the set Γ_t and the next period's level of executive constraints represented by the set Γ_{t+1}, the incumbent also chooses the amount of investment into the democratic capital stock of the society. The level of democratic capital in period t is denoted by c_t and determines the difficulty of changing the level of executive constraints. Specifically, whenever the period-t incumbent changes the level of executive constraints for tomorrow so that $\Gamma_{t+1} \neq \Gamma_t$, he incurs a cost equal to c_t. The stock of democratic capital is a state variable, because an incumbent inherits it from his predecessor.

The incumbent can determine tomorrow's stock of democratic capital by either investing or disinvesting in it. Investments (or disinvestments) are denoted by I_t so that the democratic capital evolves according to the following standard formulation:

$$c_{t+1} = (1-\delta)c_t + I_t, \tag{2.2}$$

where $\delta \in (0,1)$ is the depreciation rate. Note that the only constraint on the choice of I_t is that disinvestments cannot exceed the existing stock of democratic capital, i.e.

$$I_t \geq -c_t. \tag{2.3}$$

After the incumbent makes a policy, institutional, and investment decision, his utility in period-t can be written as

$$-d(\hat{p}_i, p_t) - I_t - c_t \tag{2.4}$$

if he chooses $\Gamma_{t+1} \neq \Gamma_t$, and as

$$-d(\hat{p}_i, p_t) - I_t \tag{2.5}$$

if he picks $\Gamma_{t+1} = \Gamma_t$, thereby not reforming the level of executive constraints.[6]

The timing of events can be summarized as follows:

- At the beginning of period t, agent $i \in \{A, B\}$ is exogenously elected according to probability q_i to become the incumbent.

[6] I have normalized the price of investment to 1.

- Taking Γ_t and c_t as given, the incumbent chooses $p_t \in \Gamma_t$, I_t, and Γ_{t+1}.
- Period t ends and payoffs are distributed.
- At the beginning of period $t+1$, the new incumbent is elected and makes the same decisions by now taking Γ_{t+1} and $c_{t+1} = (1-\delta)c_t + I_t$ as given.

Before defining and analyzing a political equilibrium of this game, I first solve the dictator's problem in order to demonstrate the crucial role electoral uncertainty plays on the agents' decisions.

2.4 The Dictatorship Solution

Studying executive constraints and the accumulation of democratic capital that makes it more difficult to change these constraints is only meaningful under political uncertainty. This is because when making their institutional and investment decisions, agents consider the possibility that policies they oppose could be enacted in the future. Moreover, unlike in Chapter 1 in which a shared preference for the public good component of policy choice allowed the dictatorial solution to serve as a benchmark, the static nature of the policy decision here makes such a comparison uninteresting. Yet, presenting the dictator's problem clears the intuition for the political problem ahead.

Suppose without loss of generality that $q_A = 1$ and $q_B = 0$ so that agent A is the dictator. In any given period t with $T-t$ more periods to live, the dictator maximizes his dynamic utility by choosing a policy p_t, tomorrow's level of executive constraints

Γ_{t+1} represented by the interval length ℓ_{t+1}, and the level of investment I_t to the stock of democratic capital c_t, taking c_t and Γ_t as given. This problem can be represented as follows:[7]

$$\max_{p_t,\ell_{t+1},I_t} \sum_{t}^{T} \beta^{t-1}[-d(\hat{p}_A, p_t) - I_t - c_t] \tag{2.6}$$

subject to

$$p_t \in \Gamma(\ell_t); \tag{2.7}$$

$$\ell_{t+1} \geq 0; \tag{2.8}$$

$$I_t \geq -c_t; \tag{2.9}$$

$$c_{t+1} = (1-\delta)c_t + I_t; \tag{2.10}$$

where the c_t term is present in the objective function only if $\ell_{t+1} \neq \ell_t$. This is the period in which the dictator pays a positive cost of institutional reform. In contrast, for all the periods t such that $\ell_{t+1} = \ell_t$, the c_t term drops out of 2.6.

If dictator A incurs the one-time cost of c_t in period t in order to get rid of any executive constraints on his policy choice, he will enjoy his ideal policy forever starting in period $t+1$. This is due to the fact that he would never impose constraints on his own decision-making in the absence of political uncertainty. On the other hand, as long as he chooses to not reform the executive constraints, he picks the closest policy to his ideal under the binding institutions.

Let $p_t^*(\ell_t)$ denote dictator A's optimal policy choice when the institutionally feasi-

[7] Note that agent A is identical to a social planner who assigns all the weight in the society's aggregate utility to agent A.

ble set is given by $\Gamma(\ell_t)$. Dictator A's institutional strategy $\rho_t^* : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that $\rho_t^*(\ell_t, c_t) = \ell_{t+1}$ and investment strategy $\gamma_t^* : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that $\gamma_t^*(\ell_t, c_t) = c_{t+1}$ constitute an equilibrium if and only if they solve his maximization problem 2.6 subject to the associated constraints 2.7-2.10 for all periods t. The following proposition describes the dictatorship equilibrium for the model with $T = 3$:[8]

Proposition 1. *A dictator never invests into the society's stock of democratic capital or tightens the executive constraints on his rule in equilibrium. Specifically, his equilibrium institutional strategy can be described as follows:*

1. *Whenever c_1 is sufficiently low and $\Gamma(\ell_1)$ is sufficiently restrictive such that*

$$c_1 \leq \beta(1+\beta)d(\hat{p}_A, p_t^*(\ell_1)) \tag{2.11}$$

   for any t, dictator A always reforms the executive constraints by choosing $\ell_t \neq \ell_1$ such that $\hat{p}_A \in \Gamma(\ell_t)$ for either $t = 2$ or $t = 3$. Otherwise, maintaining the status-quo ℓ_1 is the optimal action.

2. *Whenever 2.11 holds and δ is sufficiently high such that*

$$c_1 > \frac{\beta d(\hat{p}_A, p_t^*(\ell_1))}{1 - \beta + \beta\delta} \tag{2.12}$$

   for any t, dictator A prefers to delay the reform of executive constraints until $t = 2$ over immediate reform.

[8] All the proofs are in Appendix D.

Proposition 1 indicates that no accumulation of democratic capital takes place unless there exists some political uncertainty. An agent who is certain to be re-elected will neither willingly impose constraints on his own rule nor pay for investments into increasing the difficulty of institutional reform. Based on this result, the description of a dictator's institutional decision in equilibrium is intuitive: Executive constraints are eventually reformed to allow for the dictator's ideal policy if the policy cost of maintaining restrictive institutions justifies paying the cost of reform. However, delaying reform until the second period may be profitable for the dictator whenever depreciation is high enough to obviate the need for disinvestments. On the other hand, the dictator maintains the existing level of executive constraints if the cost of reform is forbiddingly high or if the existing constraints are not too restrictive.

The following section analyzes equilibrium under political uncertainty in the absence of a democratic capital variable. The main motivation for this section is to demonstrate how a combination of static policies and executive constraints as in the existing literature falls short of explaining the observed empirical facts.

2.5 The Benchmark Model

Consider the political model presented in Section 2.3 in the absence of a democratic capital variable. Suppose that the agents live for only two periods and that they have polarized policy preferences. Specifically, assume that if $\hat{p}_A > 0$, then $\hat{p}_B < 0$, and vice-versa. As in Section 2.4, let $p_t^i(\ell_t)$ denote the constrained-optimal policy choice of agent i in period t when the executive constraints are given by $\Gamma(\ell_t)$. Since the agents have complete information on each other's preferences, the institutional choice of the first period incumbent can be formulated as designating the policies that would be chosen by either potential incumbent in the final period.

The benchmark model can be summarized as follows: The period-1 incumbent chooses today's policy p_1 and tomorrow's level of executive constraints ℓ_2, taking ℓ_1 as given. At the beginning of period 2, a new incumbent is realized according to the fixed probability q_i for $i \in \{A, B\}$. Now taking ℓ_2 as given, the second period incumbent chooses a policy p_2.

I solve for the Subgame Perfect Nash equilibrium of this benchmark model via backward induction. Taking ℓ_2 as given, the second period incumbent i chooses $p_2 \in \Gamma(\ell_2)$ in order to maximize his stage utility $-d(\hat{p}_i, p_2)$. Then, given this optimal choice $p_2^i(\ell_2)$, the first period incumbent i's optimization problem can be written as follows:

$$\max_{p_1, \ell_2} \; - d(\hat{p}_i, p_1) - \beta[q_A d(\hat{p}_i, p_2^A(\ell_2)) + q_B d(\hat{p}_i, p_2^B(\ell_2))] \tag{2.13}$$

subject to

$$p_1 \in \Gamma(\ell_1); \tag{2.14}$$

$$\ell_2 \geq 0. \tag{2.15}$$

Incumbent i's institutional strategy $\rho^i : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\rho^i(\ell_1) = \ell_2$ constitutes a Subgame-Perfect Nash equilibrium if and only if it solves his maximization problem 2.13 subject to the associated constraints 2.14-2.15.[9] Based on the above program, the

[9]Since there is only a single institutional strategy in a two-period game, no time subscripts are needed on the rule ρ^i.

following proposition summarizes an incumbent's institutional decision in equilibrium:

Proposition 2. *In the Subgame-Perfect Nash equilibrium of the benchmark model, the optimal level of* ℓ_2 *increases as the incumbent's re-election probability increases.*

In the absence of democratic capital accumulation, Proposition 2 describes an incumbent's trade-off when deciding on the next period's level of executive constraints: As his re-election probability increases so that he becomes more confident of his ability to set policy tomorrow, the incumbent has an incentive to weaken the constraints. On the other hand, as he becomes more likely to suffer through the policy choice of his opponent, his protection motive propels him to restrict the set of permissible policies. This way, he can avert the future policy from moving too far away from his ideal.

As previously discussed in detail, this theoretical result fails to explain the observed behavior of executive constraints. Proposition 2 states that incumbents behave according to the terms that are dictated solely by their re-election prospects. However, the empirical evidence in Besley, Persson and Reynal-Querol (2014) indicates that while some incumbents make institutional decisions in line with the benchmark model's predictions, others do not.[10] Hence, there must exist other regime characteristics that interfere with how the relationship between executive constraints and electoral uncertainty works.

The following section introduces the accumulation of democratic capital as a possible explanation for this empirical observation.

[10]The introduction to Chapter 1 cites historical cases in which electorally powerful incumbents in democratic regimes did not attempt to weaken the executive constraints that were likely to bind them in the future when they returned to office.

2.6 Equilibrium with Democratic Capital

This section starts by defining an equilibrium to the model described in Section 2.3. Then, I characterize an incumbent's institutional and investment strategies in equilibrium.

2.6.1 Equilibrium Definition

I look for a Subgame Perfect Nash equilibrium of the T-period finite-horizon game described in Section 2.3, where the following two variables constitute the states of the world at any given period t: the institutionally feasible set represented by ℓ_t and the stock of democratic capital c_t. The incumbent's equilibrium strategies will be functions of these two state variables.

Given the level of executive constraints ℓ_t and the stock of democratic capital c_t, a pure policy strategy for incumbent i in period t is a rule $p_t^i : \mathbb{R}_+^2 \to \mathbb{R}$ such that $p_t^i(\ell_t, c_t) = p_t \in \Gamma(\ell_t)$. Second, a pure institutional strategy for incumbent i in period t is an executive constraint rule $\theta_t^i : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that $\theta_t^i(\ell_t, c_t) = \ell_{t+1}$ is the optimal length of the interval with midpoint zero that defines the institutionally feasible set $\Gamma(\ell_{t+1})$. Finally, a pure democratic capital strategy for incumbent i in period t is an investment rule $\gamma_t^i : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that $\gamma_t^i(\ell_t, c_t) = c_{t+1}$ yields the society's level of democratic capital in the next period.

Let $\sigma_t \equiv (\sigma_t^A, \sigma_t^B)$ denote a strategy profile for period t, where $\sigma_t^i = (p_t^i, \theta_t^i, \gamma_t^i)$ for $i = A, B$. Given ℓ_t, c_t, and the strategy profile σ_t, let $V_t^i(\ell_t, c_t)$ denote agent i's period-t payoff if he is the incumbent and let $W_t^i(\ell_t, c_t)$ denote this payoff if he is not.

Specifically, let

$$V_t^i(\ell_t, c_t) = -d(\hat{p}_i, p_t^i(\ell_t, c_t)) - [\gamma_t^i(\ell_t, c_t) - (1-\delta)c_t] - c_t, \tag{2.16}$$

where the final c_t term is present as a positive cost of institutional reform only if $\ell_{t+1} \neq \ell_t$, and let

$$W_t^i(\ell_t, c_t) = -d(\hat{p}_i, p_t^j(\ell_t, c_t)), \tag{2.17}$$

where $j \neq i$.

Given the current state of executive constraints ℓ_t and the level of democratic capital c_t, the period-t incumbent i with $T-t$ future periods to live chooses p_t, ℓ_{t+1}, and I_t in order to solve the following program:

$$\max_{p_t, \ell_{t+1}, I_t} -d(\hat{p}_i, p_t) - c_t - I_t + \sum_{t+1}^{T} \beta^t [q_i V_{t+1}^i(\ell_{t+1}, c_{t+1}) + (1-q_i) W_{t+1}^i(\ell_{t+1}, c_{t+1})] \tag{2.18}$$

subject to

$$p_t \in \Gamma(\ell_t); \tag{2.19}$$

$$\ell_{t+1} \geq 0; \tag{2.20}$$

$$I_t \geq -c_t; \tag{2.21}$$

$$c_{t+1} = (1-\delta)c_t + I_t; \tag{2.22}$$

where the c_t term is present only if $\ell_{t+1} \neq \ell_t$. Given the above program, the following defines an equilibrium of this game.[11]

Definition 1. *A strategy profile $\sigma_t = (\sigma_t^A, \sigma_t^B)$ for $t = 1, .., T$ constitutes a Subgame Perfect Nash equilibrium if and only if incumbent i's policy rule $p_t^i(\ell_t, c_t) = p_t$, executive constraint rule $\theta_t^i(\ell_t, c_t) = \ell_{t+1}$, and investment rule $\gamma_t^i(\ell_t, c_t) = c_{t+1}$ solve 2.18 subject to the associated constraints 2.19-2.22 for all periods t, state pairs $(\ell_t, c_t) \in \mathbb{R}_+^2$, and $i \in \{A, B\}$.*

Based on Definition 1, since the policy choice p_t does not interact with the choice of ℓ_{t+1} or I_t, it is separable from these decisions. Therefore, the static part of incumbent i's problem in any given period t can be written as follows:

$$\max_{p_t \in \Gamma(\ell_t)} \; - d(\hat{p}_i, p_t). \tag{2.23}$$

The solution to 2.23 implies choosing the closest policy to incumbent i's ideal in every period t from the set of permissible policies $\Gamma(\ell_t)$.

Having described an incumbent's policy choice in equilibrium, I solve for an incumbent's equilibrium institutional and investment strategies in the following section when the game lasts for three periods. The reason for focusing on three periods is that a two-period model does not allow for sufficient time to characterize an incumbent's equilibrium investment strategies. This is because the investment decision in period 1 determines the stock of democratic capital in period 2, which has an effect on the period-2 institutional decision. However, since there does not exist an institutional

[11] I suppress the dependence of the functions V_t^i and W_t^i on the strategy profile σ_t to reduce clutter.

decision in the final period of analysis, the investment choice is meaningless under a two-period model.

2.6.2 Characterization of Equilibrium

An important aspect of institutional reform in this model is that it does not depend on how radical the reform is: Once an incumbent has decided to reform the executive constraints for the next period, his optimal choice of ℓ_{t+1} does not depend on the magnitude of c_t. However, his initial decision on whether to change the level of executive constraints at all or not does depend on c_t.

Given the level of executive constraints ℓ_{t-1} in period $t-1$, let

$$\epsilon_t^i(\ell_t) = q_i d(p_t^i(\ell_t), p_t^i(\ell_{t-1})) - q_j d(p_t^j(\ell_t), p_t^j(\ell_{t-1})) \tag{2.24}$$

denote agent i's expected net policy benefit in period t from choosing ℓ_t, where $j \neq i$.[12] Specifically, if the incumbent i in period $t-1$ designates $\Gamma(\ell_t)$ for the period-t set of permissible policies, his net policy benefit from this change consists of the distance he gains, given by $d(p_t^i(\ell_t), p_t^i(\ell_{t-1}))$, by moving closer to his ideal $\hat{p}_i$ if he is re-elected with probability q_i, and the distance he loses, given by $d(p_t^j(\ell_t), p_t^j(\ell_{t-1}))$, by permitting his opponent to implement a policy closer to $\hat{p}_j$ (and hence further away from $\hat{p}_i$) with probability q_j.

Using this definition, Propositions 3 and 4 below describe an incumbent's institu-

[12]In this definition and the following analysis, I suppress the dependence of the policy rule p_t^i on c_t in order to reduce clutter.

tional and democratic capital strategies in equilibrium:[13]

Proposition 3. *Let $T = 3$. The Subgame-Perfect Nash equilibrium institutional strategies of incumbent i in each period t are described as follows:*

1. *Given ℓ_2 and c_2, the period-2 incumbent i reforms the level of executive constraints so that $\theta_2^i(\ell_2, c_2) \neq \ell_2$ whenever*

$$\frac{c_2}{\beta} \leq \epsilon_3^i(\theta_2^i(\ell_2, c_2)), \tag{2.25}$$

   where $\theta_2^i(\ell_2, c_2) = \ell_3$ is determined by

$$q_i(\hat{p}_i - p_3^i(\ell_3))\frac{d \mid p_3^i(\ell_3) \mid}{d\ell_3} + q_j(\hat{p}_i - p_3^j(\ell_3))\frac{d \mid p_3^j(\ell_3) \mid}{d\ell_3} = 0, \quad j \neq i. \tag{2.26}$$

2. *The period-t incumbent i becomes more likely to reform the executive constraints so that $\theta_t^i(\ell_t, c_t) \neq \ell_t$ for $t = 1, 2$ as his re-election probability q_i increases, the initial cost of reform c_1 decreases, and the restrictiveness of the initial constraints represented by ℓ_1 increases.*

3. *The period-1 incumbent i is more likely to prefer immediate reform in $t = 1$ over delaying reform until $t = 2$ if the depreciation rate of the democratic capital stock is low and the cost of blocking opponent j's institutional decision is high.*

The above proposition describes an incumbent's optimal institutional decisions in equilibrium. Part 1 focuses on the final period of analysis for which an institutional

[13] Since the institutional and democratic capital strategies are jointly determined, the proofs of Propositions 3 and 4 are presented together in Appendix D.

decision exists and indicates that reform is undertaken in this period if the cost is less than the expected net utility from policies that would be realized under the ℓ_3 choice of the period-2 incumbent. Specifically, the period-2 incumbent changes the executive constraints such that 2.26 is satisfied whenever the expected benefit from doing so exceeds the cost. In this model, the expected benefit of reform comes from enjoying a policy in case of re-election that is closer to the incumbent's ideal $\hat{p}_i$, whereas the expected cost consists of the opponent's potential policy choice that would be picked under weaker constraints along with the out-of-pocket expense c_2.

Part 2 of Proposition 3 describes the conditions under which we are more likely to observe an eventual reform of the executive constraints. First, a lower initial stock of democratic capital increases an incumbent's payoff from reform, thereby making reform more likely. Second, a higher probability of setting policy in the future increases an incumbent's stakes from reform, making any relaxation of the given constraints more valuable. Third, the initial restrictiveness of the executive constraints determines the cost of maintaining the status-quo, thereby making reform more likely as the incumbent becomes more constrained. Specifically, the smaller the initial interval of permissible policies, the further away the incumbent's policy choices will lie from his ideal point if status-quo is preserved, leading to a lower payoff in case of his re-election.

Finally, Part 3 of Proposition 3 indicates that once an incumbent has decided to reform the executive constraints eventually if given the office, doing this immediately will be preferred to delaying it until the next period if depreciation does not work for the incumbent's advantage. In addition, delaying is less likely to be the optimal course of action if the cost of blocking his opponent is high. The intuition for why a high blocking cost propels an incumbent to reform immediately lies in the possibility of his own re-election. If the period-1 incumbent increases the stock of democratic capital sufficiently to deter his opponent from reforming the executive constraints and finds himself re-elected in $t = 2$, this would imply that he would need to pay this now-higher cost of reform himself. Therefore, the incumbent would prefer immediate reform in order to avert this risk.

Having described an incumbent's institutional strategies, I now turn my attention to his democratic capital strategies in equilibrium:

Proposition 4. *Let $T = 3$. The Subgame-Perfect Nash equilibrium democratic capital strategies of incumbent i are described as follows:*

1. *Given ℓ_1 and c_1, if the period-1 incumbent i chooses to prevent his opponent j from potentially choosing $\theta_2^j(\theta_1^i(\ell_1, c_1), c_2) \neq \theta_1^i(\ell_1, c_1)$, his investment in the stock of democratic capital is such that $\gamma_1^i(\ell_1, c_1) = \beta\epsilon_3^j(\theta_2^j(\ell_1, c_2))$.*

2. *The period-1 incumbent i becomes more likely to block his opponent j's potential institutional decision in $t = 2$ as q_j increases, the potential policy cost of allowing $\ell_3 = \theta_2^j(\ell_2, c_2)$ given by $d(p_3^j(\theta_2^j(\ell_2, c_2)), p_3^j(\ell_2))$ increases, or the potential policy benefit of $\ell_3 = \theta_2^j(\ell_2, c_2)$ given by $d(p_3^i(\theta_2^j(\ell_2, c_2)), p_3^i(\ell_2))$ decreases, where $\ell_2 = \theta_1^i(\ell_1, c_1)$ and $c_2 = \beta\epsilon_3^j(\theta_2^j(\ell_1, c_2))$.*

Based on the characterization of equilibrium presented in Appendix D, Propositions 3 and 4 together give a complete description of the model's Subgame-Perfect

Nash equilibrium. Part 1 of Proposition 4 characterizes the amount of investment necessary to block an opponent's potential institutional decision in the second period. Since the cost of institutional reform does not vary with the extent of the reform, the period-2 incumbent would choose ℓ_3 as characterized in 2.26 *if* he would change the level of executive constraints at all. Therefore, an incumbent who wishes to block his opponent invests just enough to ensure that condition 2.25 does not hold for his opponent. Investing beyond this amount unambiguously decreases his payoff as doing so further increases c_2 without accomplishing any additional goals.

For Part 2 of Proposition 4, observe that the amount of investment necessary in period 1 to block agent j is such that

$$c_2 = \beta q_j d(p_3^j(\ell_1), p_3^j(\theta_2^j(\ell_1, c_2))) - \beta q_i d(p_3^i(\ell_1), p_3^i(\theta_2^j(\ell_1, c_2))). \tag{2.27}$$

This equation based on 2.24 implies that it becomes more expensive for incumbent i to change his opponent j's potential institutional decision in the next period as agent j's electoral power increases. This is due to the fact that agent j's stakes from changing the executive constraints increases with his political power. However, the proposition also indicates that incumbent i also becomes more likely to block his opponent as q_j increases. The reason for why a higher level of q_j can make blocking more profitable all the while increasing the cost of doing so is the fact that incumbent i's stakes from preventing his opponent also increases with q_j. Based on the result in Proposition 2 that $\theta_2^j(\ell_2, c_2)$ increases with q_j for any ℓ_2 and $c_2 \leq \beta\epsilon_3^j(\theta_2^j(\ell_2, c_2))$, a higher value of q_j implies a higher probability for incumbent i of having a period-3 policy that lies further away from his ideal. As a consequence, it becomes ever more important for incumbent i to block his opponent by investing the necessary amount. In addition, Part 2 of Proposition 4 states that blocking becomes more likely as the weaker constraints opponent j would potentially pick imply a high policy cost to incumbent i in case of agent j's election and a low policy benefit in case of his own election for period 3.

An incumbent's blocking decision is motivated by two opposing incentives, which will be called the *protection* effect and the *self-trap* effect. The protection effect propels an incumbent to invest the necessary amount in order to preserve the status-quo ℓ_2, which was set by incumbent i himself. This ensures that the period-3 policy does not diverge too far away from his ideal in case of his opponent's election. On the other hand, the self-trap effect decreases an incumbent's incentive to invest, because to the extent that he is likely to be re-elected for $t = 2$, higher stocks of democratic capital make it more difficult for him to reform ℓ_2 according to his preferences if he hasn't already done so. Investing in this scenario would amount to falling in his own trap as he ends up increasing the price of his own institutional decision. Overall, while the protection effect is the main propellant behind the accumulation of democratic capital and grows with the opponent's political power, the self-trap effect works in the opposite direction. Which one of these two effects dominates in equilibrium depends on the distribution of political power.

The analysis of equilibrium suggests that a society's stock of democratic capital introduces some inertia to executive constraints. Specifically, if the cost of reform is sufficiently high, institutional reforms we might have observed otherwise may not take place. This chapter offers differences in the stock of this capital as an explanation into the varying degrees of inertia we observe across countries. The main results described in Propositions 3 and 4 imply that the highest accumulation of democratic capital takes place in societies with the greatest political turnover. The fact that initially restrictive institutions and low levels of democratic capital increase an incumbent's payoff from reform also stresses the importance of initial conditions.

2.7 Concluding Remarks

This chapter analyzed the determinants and the persistence of strong executive constraints. I model a society's democratic capital as determining the cost of changing the level of executive constraints. The model yields an incumbent's trade-off between protecting himself from his opponent's undesirable policies and not increasing the cost of his future institutional decisions when deciding on the society's stock of democratic capital.

The model features two agents with exogenous re-election probabilities for office who decide on the current policy, the level of executive constraints for the next period, and the amount of investment into the stock of democratic capital when in office. Each agent is purely policy-motivated. Hence, executive constraints and the democratic capital stock are purely instrumental in the sense that an incumbent cares about them only to the extent that they aid or hinder his ability to implement desired policies.

There are a number of features of the model that can be built upon in future work. For example, while the exogenous re-election assumption makes the model more tractable, it is important to note that introducing voters and thereby endogenizing the election process would make the problem richer. As long as the policy-motivation assumption is kept, the incumbent would still perceive the executive constraints and the democratic capital as instrumental in this case. However, he would now have to factor the voters' policy preferences into each of these forward-looking decisions as the voters would anticipate the policy consequences of his institutional and investment decisions, and vote accordingly.

Second, the society's stock of democratic capital is modeled in this paper as a cost an incumbent incurs each time he executes a change in the institutions. This is clearly a simplification intended to capture a country's regime characteristics. The fact that it is reduced-form is a simplification for more complicated institutional structures that impose higher costs of changing institutions compared to policy. Such higher costs may be rooted in the power of the opposition, an independent judiciary, a vibrant media, or international pressure. For example, the difficulty of institutional reform can be explicitly modeled using a bargaining framework between the executive and the opposition. Further research into the root causes of institutional inertia promises to be a fruitful pursuit.

Chapter 3

Bargaining Under Institutional Challenges

3.1 Introduction

In this final chapter of the dissertation, I turn my attention to analyzing the process by which institutional reform proposals get enacted. Most existing legislative bargaining models assume that the agreed-upon allocation is final, whereas in practice, there exist mechanisms for challenging passed legislation when there is lack of sufficient consensus. Specifically, such mechanisms include popular vote requirements following insufficient majorities in the legislature. In most parliamentary systems, a bill that fails to win a certain majority of votes in the legislature can be presented to a public vote as the final arbiter.[1] For example, in a referendum in May 2011,

[1] I do not consider referenda that are constitutionally-mandated regardless of the level of consensus in the parliament. For example, all but one US states require constitutional amendments to be

Britain rejected a proposal to switch from a first-past-the-post election system to an alternative vote system. In March 2011, shortly after the fall of the Mubarek regime, Egypt approved in a widely contested referendum a series of constitutional reforms, including presidential term limits and election supervision mechanisms.

Motivated by these examples, I analyze the effect of institutional mechanisms to challenge agreed-upon legislation on the formation of these bills and the equilibrium payoffs to the parties. I start by recognizing that both exogenous factors and endogenous choices affect a party's potential influence in a given post-bargaining stage. For example, a large literature, including Matsusaka (2005a) and (2005b), documents the surge in spending on referendum campaigns. Examples of such campaigns are advertising, media coverage or political rallies. Moreover, there exists growing evidence that the public is affected by these campaigns, as documented in de Figueiredo, Ji and Kousser (2011). With a new empirical approach that attempts to deal with the endogeneity of campaign spending, the authors find that spending both for and against a proposal influences the probability of its passage in the campaigners' intended direction.[2]

Given the influence of these campaigns on voters, to what extent do the proposals introduced in a parliament reflect the parties' public vote calculus? For instance, would the Egyptian constitutional reform package include more liberal propositions

approved in a referendum regardless of the level of congressional majority.

[2]The impact of campaign spending on referenda or citizen initiative outcomes has been studied in Gerber (1999) and Broder (2000). Empirical studies, including Bowler and Donovan (1998) that have treated campaign spending as exogenous find asymmetric effects of money of referendum outcomes: While spending against a proposal decreases its chances of passage, a similar effect does not exist when spending supports the proposal. Lupia and Matsusaka (2004) provides an overview and discussion of these results.

if the liberal faction were considered a more powerful player in the subsequent referendum? Specifically, how does a referendum process in which parties campaign to influence the probability of its outcome affect the contents of a legislative proposal? Under what conditions can the parties agree on a grand bargain that would obviate a referendum? Within the context of a referendum, the main goal of this chapter is to study the consequences of a strategic post-bargaining stage on the equilibrium payoffs of the players endowed with varying degrees of "post-bargaining power".

In order to address these questions, I build a one-period legislative bargaining model in which parties bargain over a bill with single-dimensional policy and distributive rent components. After the party with the most number of seats proposes both a policy and a rent allocation, other parties simultaneously vote on the proposal. If the proposal fails to win a simple majority, it is rejected and the game ends. Otherwise, the proposal passes. In the post-bargaining stage, parties can challenge the approved bill depending on its level of support in the parliament. I model the post-bargaining stage with a referendum in which parties can challenge the bill in a public vote only if it fails to receive a supermajority in the parliament. Once the challenge stage begins, parties campaign for or against the proposal to influence its outcome. The parties' exogenous campaigning budgets characterize their post-bargaining power.

I define a political equilibrium for two and three-party parliaments and characterize it under the challenge model.[3] I show that in the presence of looming institutional challenges, surplus coalitions are possible. Moreover, measures of post-bargaining

[3]A two-party parliament can be considered as representing the outcome of a first-past-the-post election system, and a three-party parliament as the outcome of a proportional representation system.

power do not necessarily translate into higher equilibrium payoffs as the proposer party faces a trade-off between a higher probability of having its bill upheld in a post-bargaining challenge by including a "powerful" party in its coalition and proposing a bill that captures a high share of benefits for itself. In two-party parliaments, a grand bargain is more likely in equilibrium if the minority party commands a low status-quo payoff and the proposer has a large campaigning budget. Similarly, parties reach a grand bargain more easily in three-party parliaments when the smaller parties do not command high status quo payoffs or if all parties are ideologically close. Moreover, I find that the chances of a referendum are higher if the campaigning budgets of the smaller parties diverge widely. This is because in equilibrium, only the status-quo payoffs determine the proposer party's utility from a grand bargain. Campaigning budgets matter only to the extent to which the proposer party can benefit from pitting one small party against another through coalition formation. More generally, a more asymmetric distribution of post-bargaining powers within a parliament incentivizes challenge procedures to the benefit of the proposer.

Having analyzed the factors that lead a dominant party to risk subsequent institutional challenges instead of inducing unanimity, the chapter then studies the composition of simple majority coalitions in three-party parliaments. In any political equilibrium, I show that the proposer party is more likely to partner with the party that has a lower status quo payoff or a closer ideal point. On the other hand, whether a large campaigning budget makes a party the preferred coalition partner depends on the type of political equilibrium. In the referendum model, this ambiguity result is a consequence of the proposer party's essential trade-off that defines its decision-making: Although a rich partner and a poor opponent is desirable for increasing the probability that its bill is upheld, it comes at the expense of higher concessions to the rich partner in the bargaining stage. Which one of these effects dominates in equilibrium depends on the parameters of the model.

3.2 Related Literature

Building on the seminal work of Baron and Ferejohn (1989) and the uniqueness of payoffs result proved in Eraslan (2002), numerous models study the equilibrium consequences of different sources of bargaining power by treating the agreed-upon allocation as the final outcome. Some of these papers include Kalandrakis (2006) who studies proposal rights, McCarty (2000) who studies proposal and veto rights, Snyder, Ting and Ansolabehere (2005) who study weighted voting, and Yildirim (2007) who studies endogenous proposal power. Another branch of this literature studies bargaining models with stochastic surplus to be divided and includes Eraslan and Merlo (2002) and Diermeier, Eraslan and Merlo (2002). In addition, dynamic bargaining models such as Kalandrakis (2004), Duggan and Kalandrakis (2012), and Bowen, Chen and Eraslan (2014) consider situations in which the agreed-upon allocation becomes the new status-quo in the next bargaining period. However, these papers do not study institutions outside of the bargaining environment through which the agreed-upon outcome can be challenged. Veto-player models such as Winter (1996) are an exception for incorporating a post-bargaining stage in which bargaining outcomes can be overturned. Another example is Powell (1996), who considers a bargaining model in which players can impose outside settlements to capture the whole pie, but this happens with pre-determined probabilities. In contrast to these exogenous sources of bargaining power, this chapter introduces a new source of bargaining power that is generated from post-bargaining behavior.

The vote of confidence mechanism in legislatures, studied in Diermeier and Feddersen (1998), is an example of a post-bargaining institution that affects the bargaining equilibrium. The authors show that the existence of such a mechanism decreases the price of building coalitions in the legislature and results in equilibrium coalitions that are more cohesive and rewarded more handsomely. Similar to the veto procedure, their study is relevant to this chapter through its explicit recognition of post-bargaining institutions directly affecting the outcomes of the bargaining game.

One of the main predictions of this chapter is the formation of surplus coalitions even though minimum winning coalitions would be sufficient for the bills to formally pass in the legislature. Even though minimum winning coalitions were the main prediction of the baseline model of Baron and Ferejohn (1989), other papers have studied environments where this prediction fails to hold. For example, Goreclose and Snyder (1996) show that equilibrium coalitions will exhibit surplus members because such coalitions will be cheaper than minimum winning ones when certain bargaining protocol conditions are met.

The institutions of direct democracy, represented by the post-bargaining referendum option in this chapter, has been studied by both economists and political scientists from different angles. Romer and Rosenthal (1979) is one of the first models that deviate from the Downsian median voter prediction to study the voter's choice between the status quo policy and some alternative proposed by a bureaucrat with agenda-setting power. Using the level of expenditures as the policy to be decided upon, they show that the actual level of expenditures will be at least as great as the one predicted in the Downsian model. Lupia and Matsusaka (2004) provide an overview of the political science literature with a focus on the effects of campaign money on the results of direct democracy exercises.

There also exists a large political science literature on the domestic ratification of international treaties using the two-level games approach, building on the seminal insight of Putnam (1988) that a smaller set of propositions that could get domestic approval increases the bargaining power of the negotiator at the international stage. Other relevant papers on two-level games include Iida (1996), Haller and Holden (1997), and Humphreys (2007). Although the sequence of the moves are similar to the referendum model, with a public vote following a bargaining stage, this chapter models the players with an eye toward the same public vote constraint as opposed to different domestic constituencies. Moreover, the constraint in the referendum model is not set exogenously by the median legislator or the median voter's ideal point, but can be influenced through endogenous campaign spending.

The main results of this chapter have implications for public financing of issue campaigns. Papers such as Coate (2004) and Ashworth (2006) study the welfare effects of private campaign finance by interest groups. Since I do not model interest groups, this paper is silent on the impact of private campaign contributions. However, comparative statics on the parties' exogenous campaigning budgets yield implications of public campaign finance for observed legislative outcomes.

Finally, this chapter models the post-bargaining referendum option as a contest between the bargaining parties and therefore draws upon many results in contest theory. The most relevant of these theoretical papers are Baik (2008), who characterizes the equilibrium in contests with group-specific prizes, and Skaperdas and Vaidya (2012) who show how a Tullock contest function, which my model uses, can proxy voter behavior in referendums. Other relevant papers in contest theory include Dixit (1987), Hillman and Riley (1989) and Skaperdas (1996).

The rest of the chapter is organized as follows: Section 3.3 introduces the model and defines a political equilibrium. Sections 3.4 and 3.5 respectively analyze equilibrium behavior in two and three-party parliaments. Section 3.6 discusses the implications of the equilibrium results on campaign finance policy and concludes.

3.3 The Model

I consider a situation of one-period legislative bargaining over a bill that consists of ideology and distributive components, followed by a referendum if the number of votes in the parliament falls within an institutionally designated interval.

Let N denote the set of parties and $|N|$ the number of parties in the parliament. In this chapter, only parliaments of two and three parties will be considered. The model consists of two stages: the bargaining stage and the challenge stage. In the bargaining stage, the party with the most number of seats proposes a bill and the other party (or parties) votes on it. In a three-party parliament, I assume that the two non-proposer parties vote simultaneously on the bill. Let $x \in [0, 1]$ represent the ideological component of the proposal and let $\hat{x}_k$ denote party k's ideal ideological point. In addition, let y represent the proposed allocation of rents from the feasible set

$$Y = \{y : \sum_{k=1}^{|N|} y_k \leq 1 \text{ and } y_k \geq 0 \; \forall \; k\}, \tag{3.1}$$

where the fixed sum of rents is given by unity and y_k denotes party k's share. Hence, a proposal can be represented by $z \equiv (x, y) \in [0, 1] \times Y$. When the proposal is introduced to the parliament, there exists a status-quo bill $s \equiv (q, y^q)$, where $q \in [0, 1]$ denotes its ideological component and $y^q \in Y$ its rent allocation. I assume that party k's preferences over a bill are represented by the quasi-linear utility function

$$u_k(z) = -(x - \hat{x}_k)^2 + \alpha y_k, \tag{3.2}$$

where $\alpha \in (0, 1)$ is some fixed weight.

After the proposer party makes an offer $z \in [0, 1] \times Y$ and the other party (or parties) votes on it, the proposal is accepted or rejected according to the following criteria: Let $\bar{k}(z)$ denote the number of parties other than the proposer who support the bill z. If $\bar{k}(z) = |N| - 1$, the proposal z is unanimously accepted and becomes the law with no subsequent challenges. If $\bar{k}(z) = 0$, the bill is automatically rejected in a three-party parliament. On the other hand, rejection without a challenge is not feasible in two-party parliaments, since the proposer party always commands a simple majority. Finally, if $\bar{k}(z) = |N| - 2$, the proposal is temporarily accepted in the parliament to be challenged in a referendum. Any proposal that survives the challenge becomes the law.[4] [5]

If the proposal passes in the parliament without unanimous support, the dissenting party takes the bill to a referendum. I describe this challenge stage as a two-candidate competition in which the candidates are the proposal z and the status quo s. Before the referendum takes place, each party k simultaneously chooses a position $t \in \{Z, S\}$ and an irreversible campaign spending amount $c \geq 0$ to influence the voters (who will not be explicitly modeled). Position Z indicates a preference for the public acceptance of the proposal (yes vote on the referendum) and position S indicates a preference for its failure (no vote on the referendum).

Each party k is allocated an exogenously given campaigning budget $w_k \in [0, 1]$.[6] Upon observing the campaigns of each group, the public votes on the proposal in a referendum. If the proposal wins a simple majority of the public vote, it becomes the law. Otherwise, the status-quo bill prevails and all parties receive their status-quo payoffs. I assume that all the parameters of the model are common knowledge.

I model the referendum as a contest between the positions Z and S in which their winning prize is given respectively by z and s. Hence, the winning prize constitutes a public good within each group of parties. Let $C_t(z)$ denote the total campaign spending of parties aligned with position t when the proposed bill is z and

[4] This acceptance criteria represents the following general rule in parliamentary systems for important legislation or constitutional amendment proposals. Let $\bar{k}$ denote the number of supportive legislators. If $\bar{k} \leq \frac{|N|-1}{2}$, where $|N|$ is odd, the proposal fails to win a simple majority and fails. If $\bar{k} \geq \lambda(|N|-1)$, where $\lambda \in (\frac{1}{2}, 1)$, it is accepted without a referendum. Finally, for all $\bar{k} \in (\frac{|N|-1}{2}, \lambda(|N|-1))$, the proposal becomes law only if it is accepted in a referendum. Here, λ represents the supermajority parameter for the parliament.

[5] In a three party parliament, I assume without loss of generality that no party commands a majority of the seats and that two parties together cannot control a supermajority.

[6] Although private interest groups play an important role in financing referendum campaigns, I do not model them here in the interest of keeping the analysis tractable.

let $p_t(C_Z(z), C_S(z))$ denote the probability that position t wins the referendum. I assume that the contest success function takes the Tullock lottery form so that the probability of winning for a party aligned with position t is given by

$$p_t(C_Z(z), C_S(z)) = \begin{cases} \frac{C_t(z)}{C_Z(z)+C_S(z)} & \text{if} \quad C_Z(z) + C_S(z) > 0 \\ \\ \frac{1}{2} & \text{if} \quad C_Z(z) = C_S(z) = 0 \end{cases} \tag{3.3}$$

for $t = Z, S$ and proposal z. The above Tullock specification assumes that neither party has an inherent advantage in the contest. Moreover, it implies that a position's winning probability is increasing in the spending of the parties aligned with it and decreasing in the spending for the other position.

The sequence of events can be summarized as follows:

- The proposer party offers a bill z to the parliament.
- The other party (or parties) votes on z. If the vote(s) is such that the decision is not final, the challenge stage begins.
- Each party simultaneously and independently chooses a position t and an irreversible campaign spending c for the referendum.
- The public votes in the referendum. If the proposal wins a simple majority of the public vote, it passes and becomes the law. If not, all players receive their status quo payoffs.

A pure bargaining strategy for party k consists of a proposal $z \in [0, 1] \times Y$ if k is the proposer party, and an acceptance rule $a_k : [0, 1] \times Y \rightarrow \{0, 1\}$ for the non-proposer parties k such that $a_k(z) = 0$ indicates rejection of the proposal z and $a_k(z) = 1$ indicates its acceptance.[7] In addition, a pure challenge strategy for party k consists of the following elements: a position rule $\rho_k : [0,1] \times Y \to \{Z, S\}$ such that $\rho_k(z) = t$ indicates that party k has chosen position t for the referendum; and a campaign spending rule $\zeta_k : [0,1] \times Y \to [0, w_k]$ such that $\zeta_k(z) = c$ yields the amount party k spends on his chosen position's campaign. Specifically, $\rho_k(z) = t$ indicates that party k spends an amount $c = \zeta_k(z)$ for position t. A party jointly chooses its position and campaign spending amount.

Without loss of generality, fix party 1 as the proposer party. Let $\sigma \equiv (\sigma_1, \{\sigma_k\}_{k=2}^{|N|})$ denote a strategy profile, where $\sigma_1 = (z, \rho_1, \zeta_1)$ for the proposer party and $\sigma_k = (a_k, \rho_k, \zeta_k)$ for the non-proposer party (or parties) $k \neq 1$.

Let $N_Z = \{k \in N : \rho_k(z) = Z\}$ and $N_S = \{k \in N : \rho_k(z) = S\}$ respectively denote the set of parties that align themselves with positions Z and S. Then, given a proposal z, the total campaign spending of each party group N_t can be written as

$$C_t(z) = \sum_{k \in N_t} \zeta_k(z). \tag{3.4}$$

Given the equilibrium behavior of every other player, a political equilibrium to this game consists of optimal party strategies during both the bargaining and the challenge stages. Through backward induction, I solve for the Subgame-Perfect Nash equilibrium of this model, which is defined below:

Definition 1. *A strategy profile* $(\sigma_1, \{\sigma_k\}_{k=2}^{|N|})$ *constitutes a political equilibrium if and*

[7] I assume that a party votes to accept a proposal when indifferent.

only if the following conditions are satisfied:

(E1) Given z and $a_k(z)$ for $k \neq 1$ from the bargaining stage, and other parties' challenge strategies ρ_{-k} and ζ_{-k}, party k's position rule $\rho_k(z) = t$ and campaign spending rule $\zeta_k(z) = c$ solve

$$\max_{t \in \{Z,S\}, c \in [0, w_k]} u_k(s) + p_Z\left(\sum_{k \in N_t} c + \zeta_{-k}(z), C_{-t}(z)\right)[u_k(z) - u_k(s)] - c. \quad (3.5)$$

(E2) For any given proposal z, let

$$V_k(z;\sigma) = u_k(s) + p_Z(C_{\rho_k(z)}(z), C_{-\rho_k(z)}(z))[u_k(z) - u_k(s)] - \zeta_k(z) \quad (3.6)$$

denote party k's maximized expected payoff from the referendum when each party would be following its equilibrium challenge strategies. Then,

- *If $|N| = 2$, or $|N| = 3$ and $a_{-k}(z) = 1$, $a_k(z) = 1$ if and only if $u_k(z) \geq V_k(z;\sigma)$;*
- *If $|N| = 3$ and $a_{-k}(z) = 0$, $a_k(z) = 1$ if and only if $V_k(z;\sigma) \geq u_k(s)$.*

(E3) Party 1's proposal z solves

$$\max_{z \in [0,1] \times Y} u_1(s) + p_Z(C_Z(z), C_S(z)) \cdot [u_1(z) - u_1(s)] - \zeta_1(z). \quad (3.7)$$

Condition (E1) requires that each party's position and campaign spending rules jointly maximize its expected payoff from the referendum. Condition (E2) rules out the use of weakly dominated strategies by the non-proposer party (or parties) during legislative voting. It requires that an acceptance vote is given to a proposal if and only if it is weakly preferred to voting to reject it. Finally, condition (E3) requires that given the subsequent optimal acceptance, position, and campaign spending rules of all the parties, the proposer party 1 makes an offer that maximizes its expected payoff. Before the bargaining stage begins, the referendum probabilities p_t for $t = Z, S$ are within the control of party 1. Specifically, the proposer can induce any possible outcome by making the right offer.

Given the existence of equilibria in contests that describe the challenge stage of this model and the existence of a bargaining equilibrium for any profile of challenge strategies, a political equilibrium exists. In the following sections, I characterize the pure strategy Subgame Perfect Nash equilibria of this model respectively for parliaments of two and three parties.

3.4 Two-Party Parliaments

Two-party parliaments can be thought of as representing the outcome of a first-past-the-post election system. In this context, I assume that the proposer party 1 controls a simple majority, but not a supermajority, of the seats so that it needs the approval of the smaller party 2 in order to avoid a challenge stage.

I solve for the political equilibrium in a two-party parliament through backward induction. First, consider the parties' equilibrium challenge strategies. If the game reaches this stage, the parties' position choices for the referendum are trivial: On the equilibrium path, party 1 never campaigns against its own proposal so that $\rho_1(z) = Z$ always holds for any given proposal z. Similarly, if party 2 preferred a yes vote on the referendum, it would have accepted the proposal z during bargaining in order to secure a sure outcome and not incur campaigning costs. Hence, $\rho_2(z) = S$ always holds as well on the equilibrium path.

Given the equilibrium position rules described in the above paragraph, the optimal campaign spending of the two parties for any given proposal z from the bargaining stage are given by

$$\zeta_1(z) \in \arg\max_{c\in[0,w_1]} \frac{c}{c+\zeta_2(z)} u_1(z) + \frac{\zeta_2(z)}{c+\zeta_2(z)} u_1(s) - c; \tag{3.8}$$

$$\zeta_2(z) \in \arg\max_{c\in[0,w_2]} \frac{\zeta_1(z)}{\zeta_1(z)+c} u_2(z) + \frac{c}{\zeta_1(z)+c} u_2(s) - c. \tag{3.9}$$

For a more concise exposition in the following analysis, let

$$\epsilon_k(z) = |u_k(z) - u_k(s)| \tag{3.10}$$

represent party k's stake from the challenge stage for any given proposal z, given by the difference in its utility from the two potential outcomes z and s. Based on 3.8 and 3.9, the following lemma describes how the parties' equilibrium campaign spendings respond to the bargaining outcome:[8]

Lemma 1. *Let z and z' be two proposals such that $\epsilon_k(z) \geq \epsilon_k(z')$ for party $k \in \{1, 2\}$. Then, $\zeta_k(z) \geq \zeta_k(z')$.*

[8]All proofs are in Appendix E.

Lemma 1 states that each party spends more in equilibrium as its stake from the referendum increases. For example, if one of two proposals implies a much lower payoff relative to the status-quo for party 2, then party 2 would fight harder for the failure of this proposal in the referendum. The larger difference between the winning and the losing prizes justifies a higher amount of equilibrium campaign spending compared to the proposal with the lower stakes.

In the following analysis, I first present the general characteristics of a political equilibrium in Proposition 1. Then, I focus on the parameter values that make a political equilibrium in which the challenge stage is reached on the equilibrium path more likely to be observed than one in which the parties settle in the parliament.

Proposition 1. *In the political equilibrium of a two-party parliament,*

1. *The acceptance rule of party 2 is characterized as follows:*
   - *Party 2 rejects any offer z for which $\epsilon_1(z)$ and $\epsilon_2(z)$ are such that $\zeta_2(z) = w_2$;*
   - *For the range of proposals z for which $\epsilon_1(z)$ and $\epsilon_2(z)$ would imply a challenge stage equilibrium with $\zeta_1(z) = w_1$ and $\zeta_2(z) < w_2$ if rejected, party 2 accepts any offer z such that $u_2(z) + w_1 \geq u_2(s)$;*
   - *For the range of proposals z for which the implied challenge stage equilibrium is an interior one, party 2 accepts any offer z that yields $u_2(z) \geq u_2(s)$.*
2. *If party 1 chooses to induce unanimity, it proposes z such that $u_2(z) = u_2(s) -$*

w_1, where

- *z features equal compromise on ideology, i.e. $x = \frac{\hat{x}_1+\hat{x}_2}{2}$;*
- *The difference between the parties' rent shares, i.e. $y_1 - y_2$, increases as $u_2(s)$ decreases or w_1 increases;*
- *For low (high) values of α, party 1 may choose $y_1 = 0$ ($y_1 = 1$) and x closer to $\hat{x}_1$ ($\hat{x}_2$).*

3. *If party 1 chooses to induce a referendum, it becomes more likely to do so by proposing $z = (\hat{x}_1, 1, 0)$ as opposed to any other proposal that yields a higher utility for party 2 as the two parties diverge ideologically.*

The first part of Proposition 1 characterizes party 2's equilibrium bargaining strategy. It indicates that any proposal that implies a sufficiently high stake for party 2 (either due to a high status-quo payoff, a very unfavorable proposal, or both) so that it would fight by spending its entire campaigning budget in a subsequent challenge will be rejected by party 2. On the other hand, party 2 may be willing to settle for proposals that involve relatively lower stakes. For instance, if the proposal is such that neither party's stake would justify exhausting its whole budget in a potential campaign, the typical criteria that party 2 accepts any proposal that leaves it at least as well-off as the status-quo applies. However, there may also exist situations in which the threat of a challenge allows the proposer to extract a surplus from party 2's status-quo payoff in a settlement. Specifically, if the parameters of the model are not too extreme so that party 2 commands a sufficiently low status-quo and w_1 is not too large, party 2 will settle for less than its status-quo payoff. This is due to the threat a looming challenge stage poses for itself. With party 1 willing to exhaust its budget to defend its relatively higher stakes from the proposal, party 2's meagre winning prize would not justify its counter campaign spending to defend the status-quo in this situation. Therefore, it is willing to pay a premium to party 1 in order to avoid this expensive challenge.

The second part of the proposition describes the optimal way to induce unanimity from party 1's point of view. The proposition states that party 1 would extract a surplus of w_1 from party 2 in a settlement, reflecting the threat discussed in the above paragraph. The optimal proposal to induce this settlement involves an equal ideological compromise between the parties. However, if the status-quo ideology is such that party 2 would gain from this compromise, party 1 extracts these gains away in the form of a higher rent share.

The final part of the proposition focuses on the type of challenge equilibrium that would be preferred by party 1. The analysis indicates that the optimal proposal to induce a challenge in which party 2 exhausts its campaigning budget is the one that maximizes party 1's winning prize, given by $z = (\hat{x}_1, 1, 0)$. This is due to the fact that the probability of winning for party 1 is not affected by how much further party 2's stake increases if party 2 is already spending its entire budget. For all other types of challenge stage equilibria in which $\zeta_2(z) < w_2$, the proposer faces the following trade-off: Even though a more favorable proposal for itself increases party 1's winning prize, this comes at the expense of decreasing its winning probability as party 2 fights more aggressively by spending more. As $\hat{x}_1$ and $\hat{x}_2$ diverge, party 1's expected payoff from this challenge may decrease sufficiently that a proposal short of $z = (\hat{x}_1, 1, 0)$ is no longer justified. More specifically, as the value of $\epsilon_2(z)$ increases due to this divergence, leading to higher spending by party 2, the proposal compromise that was made in the hopes of putting a check on party 2's spending no longer pays off. In this situation, party 1 would be better-off offering $x = \hat{x}_1$ with all the rent allocated to itself, thereby provoking an all-out fight with $\zeta_1(z) = w_1$ and $\zeta_2(z) = w_2$.

Having described party 1's incentives in choosing how best to realize a unanimity outcome in the parliament or to induce a challenge, it remains an open question which option party 1 will prefer. The following proposition takes up this task:

Proposition 2. *Party 1 is more likely to prefer the unanimity outcome over a challenge for lower values of $u_2(s)$ and higher w_1. A lower w_2 incentivizes unanimity only if $w_2 > \alpha - u_1(s)$.*

The intuition for why a smaller status-quo payoff for party 2 unambiguously contributes to a higher likelihood of observing unanimity is straight-forward: Since party 1 offers $u_2(z) = u_2(s) - w_1$ to party 2 in order to get its acceptance, a lower $u_2(s)$ increases its sure payoff from the settlement. On the other hand, while a higher w_1 may contribute to a higher probability of winning for party 1 in a particular challenge, it also increases its unanimity payoff as w_1 is extracted from party 2. In equilibrium, the effect of w_1 on its unanimity payoff dominates the challenge stage effect, yielding the result in Proposition 2.

The conditional result in Proposition 2 on how w_2 affects party 1's incentives between a settlement and a challenge illustrates another trade-off. In a challenge stage equilibrium with $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$, changes in w_2 only affect party 1's probability of winning in the referendum. On the other hand, if the equilibrium is such that $\zeta_1(z) < w_1$ and $\zeta_2(z) = w_2$, changes in w_2 affect not only party 1's probability of winning, but also its campaign spending. Specifically, a higher w_2 unambiguously decreases party 1's winning probability in this equilibrium, while it also decreases $\zeta_1(z)$ when the condition in Proposition 2 holds. When this is true, the marginal effect of lower campaign spending on party 1's expected payoff from this challenge dominates the marginal effect of a lower winning probability, resulting in an increase in party 1's expected challenge payoff. Thus, in this challenge stage equilibrium, sufficiently higher values of w_2 do not act as threat instruments due to their indirect effect on party 1's campaign spending.

Based on this analysis, we would expect to observe a proposer party with a high campaigning budget work towards achieving unanimity by buying the smaller party out. In contrast, a small party would act more aggressively by shunning a settlement if the stakes from the proposed bill are high enough. As the smaller party's budget grows, this can initially act as a threat and therefore encourage unanimity. However, this effect may be reversed once a threshold is crossed. At this point, the smaller party's budget starts to constitute an impediment to settlement.

3.5 Three-Party Parliaments

Studying a three-party parliament offers richer dynamics on coalition formation and incentives for a grand bargain in a non-cooperative framework than the two-party setting allowed. In this section, I assume that neither party controls a simple majority of the seats and that two parties together do not command a supermajority. Therefore, at least two parties must agree in order for a bill to pass in the parliament. A bill that has passed in the parliament with votes short of unanimity moves to the challenge stage. To abstract away from potential informational advantages, I assume that after party 1 makes an offer, the other two parties vote on it simultaneously.

When making a proposal, party 1 can induce one of the following four general outcomes: A grand bargain with unanimous agreement among all the three parties; rejection in the parliament; a challenge stage with party 2 as its partner and party 3 in the opposition; or a challenge stage with party 3 as its partner and party 2 in the opposition. Looking for a political equilibrium in a three-party parliament involves solving for the optimal offers that would induce each of the alternative outcomes and comparing party 1's maximum expected payoffs from those outcomes.

Baik (2008) characterizes the pure-strategy Nash equilibrium of group contests in which the winning prize is a public good within each group. Since the winning probability in the referendum is a function of each party group's *total* campaign spending, this characterization applies to the equilibrium of the challenge stage in this model. Specifically, since there are always two parties aligned with position Z in a challenge, the proposal z, which is the winning prize for members of group N_Z, constitutes a public good within this group.

To start characterizing the equilibrium of the challenge stage, first consider the parties' position choices for a given proposal z. As in the two-party case, it can never be optimal for the proposer to take a stand against its own bill so that we have $\rho_1(z) = Z$ on the equilibrium path for any given z. In order to have reached the challenge stage, it must have been the case that one party voted for the bill and one against it in the parliament. Let h and j denote these two non-proposer parties such that $a_h(z) = 1$ and $a_j(z) = 0$. If party h preferred a no vote on the referendum, it would have voted to reject the proposal in the bargaining stage, leading to its defeat and thereby avoiding a costly and risky referendum. Therefore, $\rho_h(z) = Z$ on the equilibrium path. Similarly, if party j preferred a yes vote on the referendum, it would have voted to accept the proposal during bargaining, leading to a unanimous agreement on z. Hence, it must be the case that $\rho_j(z) = S$ on the equilibrium path. Therefore, party h for whom $a_h(z) = 1$ becomes party 1's partner in the challenge stage and party j for whom $a_j(z) = 0$ becomes its opponent.

In the challenge stage, each group N_t, $t \in \{Z, S\}$, decides on a total campaign spending $C = C_t(z)$, where $C_t(z)$ is as defined in 3.4. The members of a group do not act cooperatively; instead, campaign spending choices are made independently. For a given proposal z and the total campaign spending of group N_S given by $C_S(z) = \zeta_j(z)$, let $C_Z^1(z)$ denote the best response total campaign spending of group N_Z to $C_S(z)$ from the perspective of party 1 and let $C_Z^h(z)$ denote the same best response from the perspective of its partner party h. Specifically, define $C_Z^1(z)$ and $C_Z^h(z)$ such that

$$C_Z^1(z) \in \underset{C \in [0, w_1 + w_h]}{\arg\max} \frac{C}{C + C_S(z)} u_1(z) + \frac{C_S(z)}{C + C_S(z)} u_1(s) - \zeta_1(z); \tag{3.11}$$

$$C_Z^h(z) \in \underset{C \in [0, w_1 + w_h]}{\arg\max} \frac{C}{C + C_S(z)} u_h(z) + \frac{C_S(z)}{C + C_S(z)} u_h(s) - \zeta_h(z). \tag{3.12}$$

As long as the proposal z is such that $\epsilon_1(z) \neq \epsilon_h(z)$, party 1 and its partner have different opinions as to how they should best respond to $C_S(z)$. Moreover, since the winning prize z is a public good for them, the decision on how the burden of the total spending $C_Z(z) =$ will be shared in equilibrium is not trivial.

The following lemma, based on Baik (2008), characterizes how the total campaign spending $C_Z(z)$ of group N_Z is determined and its burden is shared among parties 1 and h in a Nash equilibrium. This lemma will then be used to characterize the challenge stage equilibrium.

Lemma 2. *Suppose the proposal z is such that $\epsilon_1(z) \geq \epsilon_h(z) > 0$. Then, taking the total campaign spending $C_S(z) = \zeta_j(z)$ of group N_S as given, parties 1 and h choose their total equilibrium campaign spending $C_Z(z)$ and its allocation between $\zeta_1(z)$ and $\zeta_h(z)$ as follows:*

1. *If $C_Z^1(z) \leq w_1$, then $C_Z(z) = \zeta_1(z) = C_Z^1(z)$ and $\zeta_h(z) = 0$.*

2. *If $C_Z^h(z) \geq w_1 + w_h$, then $C_Z(z) = w_1 + w_h$, $\zeta_1(z) = w_1$, and $\zeta_h(z) = w_h$.*

3. *If $C_Z^1(z) > w_1$ and $C_Z^h(z) \leq w_1 + w_h$, then $C_Z(z) = max\{C_Z^h(z), w_1\}$, $\zeta_1(z) = w_1$, and $\zeta_h(z) = max\{0, C_Z^h(z) - w_1\}$.*

Lemma 2 provides a full characterization of the equilibrium campaign spending decisions of the members of group N_Z. To gain some intuition, first note that the party with the higher stake from a challenge, determined by the proposal z from the bargaining stage, will have a higher total campaign spending best response to group N_S than its opponent. Part 1 of the lemma indicates that if the party with the higher stake can afford its best response total campaign spending using only its own resources, then it is the only member of group N_Z that contributes to the campaign in equilibrium; its partner free-rides on its spending. This campaign more than meets the partner's needs, obviating any spending on the partner's part. On the other hand, if the total resources of the group cannot cover even the lower best response of the partner, then part 2 of the lemma indicates that each member exhausts its budget in equilibrium. There exists no free-riding in this situation. Finally, if the party with the higher stake cannot afford its best response total campaign spending with its own resources but the partner's lower best response can be met with the total group budget, then the higher-stake party spends its entire budget on the campaign while its partner contributes the difference (if the difference is positive). In this scenario, the partner is at best a partial free-rider on the higher-stake party's campaign spending.

In brief, Lemma 2 shows that unless the stakes from a challenge are sufficiently high for both members of group N_Z, the party with the lower stake free-rides on its partner's campaign spending that contributes positively to its probability of winning in the referendum. The following lemma uses the results of Lemma 2 in order to describe the general properties of a challenge stage equilibrium, which requires that group N_Z is in equilibrium and that both groups are best-responding to each other:

Lemma 3. *Let z and z' be two proposals such that $\epsilon_k(z) \geq \epsilon_k(z')$ for party $k \in \{1, 2, 3\}$. Then, $\zeta_k(z) \geq \zeta_k(z')$ in equilibrium. Moreover, for any given proposal z, the condition $\epsilon_1(z) \geq \epsilon_h(z)$ needs to hold in order for party $h \in N_Z$ to free-ride on $\zeta_1(z)$ in a challenge stage equilibrium.*

Lemmas 2 and 3 together describe the properties of a challenge equilibrium for any proposal z from the bargaining stage. Based on this challenge equilibrium, the political equilibrium of the model can be solved for via backward induction. The following propositions present general results on a political equilibrium. Following the same order of analysis as in the previous section, I study the structure of proposals that would respectively induce a grand bargain in the parliament and a subsequent challenge. Then, I focus in the remainder of the section on the conditions that make a grand bargain among the three parties more likely to be observed on the equilibrium path than a challenge.

Proposition 3. *In the political equilibrium of a three-party parliament, the following are true about inducing a grand bargain among the parties:*

1. *Any proposal z that would imply a challenge stage equilibrium with $\zeta_j(z) = w_j$ for $j \in N_S$ if rejected will move to a challenge.*

2. *In a unanimous agreement on a proposal z that would otherwise lead to a challenge with free-riding in group N_Z, the party who would have been the free-rider partner is punished.*

3. *In party 1's optimal unanimity-inducing offer z, its rent share y_1 increases in*

y_1^q, w_1, and $(q - \hat{x}_k)$ for $k = 2, 3$. Furthermore, its unanimity payoff $u_1(z)$ increases as the three parties get ideologically closer.

The first part of Proposition 3 presents a result on the structure of proposals on which a grand bargain is achievable. Specifically, it indicates that if an offer involves very high stakes for at least one party, either due to a high status-quo payoff or an unfavorable treatment in the proposal for that party, such that it would fight with all its budget in a potential challenge, unanimity is impossible to achieve. For this party, its certain payoff from unanimity is not high enough to justify foregoing the chance of regaining its status-quo payoff in a challenge. This mirrors the result in part 1 of Proposition 1 for a two-party parliament. In both cases, parties that have too much to lose from a proposal will not settle.

Part 2 of Proposition 3 suggests that a proposal z on which a grand bargain is possible reflects the division of $C_Z(z)$ among parties 1 and $h \in N_Z$ that would be observed if z was instead rejected. For example, the proof shows that if an offer z implies a challenge stage equilibrium in which $\zeta_1(z) = w_1$ and $\zeta_h(z) = 0$, party 1 extracts a premium from party $h \in \{2, 3\}$ equal to w_1 in a grand bargain. Likewise, if the opposite is true, party 1 needs to offer party h a premium of w_h in order to persuade it to join in the agreement.

The final part of Proposition 3 characterizes the properties of the optimal offer for party 1 that would induce unanimity. Not surprisingly, we observe that party 1 captures a higher share of the surplus as it becomes a more powerful player, either due to a higher status-quo or a higher campaigning budget. The intuition for these effects is as follows: A higher status-quo rent share for party 1 means that the other parties command less, thereby decreasing the amount they need to be compensated for in a grand bargain. Likewise, the more non-proposer parties are away from their ideal ideological points in the status-quo, the lower the compensation they require. On the effect of w_1 on y_1, the proof shows that party 1's optimal unanimity-inducing offer z is such that if rejected, it would lead to a challenge equilibrium with $\zeta_1(z) = w_1$. Thus, w_1 can be interpreted as party 1's reward for making an offer that "saves" the non-proposer parties the spending on their groups' campaigns. Nonetheless, party 1 needs to compensate them for their ideological loss in the form of higher rent shares in proposal z. Therefore, the results indicate that an ideologically-divided parliament always hurts party 1 in a grand bargain.

Having studied the structure of a unanimous agreement in a three-party parliament and how to best get there from party 1's point of view, the following proposition focuses on the same questions for a referendum:

Proposition 4. *In the political equilibrium of a three-party parliament, the following are true about inducing a challenge with party h as the partner and party j as the opponent of party 1:*

1. *For any challenge-inducing proposal z, party 1's expected payoff from a challenge increases as y_h^q decreases, $(q - \hat{x}_h)^2$ increases, and $\hat{x}_1$ and $\hat{x}_h$ get closer.*

2. *For any challenge-inducing proposal z for which $\zeta_h(z) > 0$, a higher w_h decreases party 1's expected payoff from the challenge if w_h and $u_1(s)$ are sufficiently high;*

3. *All else constant, party 1 prefers to partner with party 2 instead of party 3 if*

– $u_2(s) \leq u_3(s)$;

– $w_2 > w_3$ *for any proposal that implies a challenge stage equilibrium with* $\zeta_h(z) = 0$;

– $w_2 \leq w_3$ *whenever* w_h *and* $u_1(s)$ *are sufficiently high, and* $w_2 > w_3$ *otherwise, for any proposal* z *that implies a challenge stage equilibrium with* $\zeta_h(z) > 0$.

The results in Proposition 4 illustrate party 1's incentives when deciding on the identity of its partner in a challenge. First, the proposition states that it necessarily increases party 1's expected payoff from a challenge if its partner has a lower status-quo payoff. This is due to the fact that a party always requires at least its status-quo payoff in order to become party 1's partner regardless of whether it will contribute to group N_Z's campaign spending or become a free-rider in equilibrium. Thus, a lower status-quo payoff makes it more likely for a party to be designated as party 1's partner in a challenge-inducing proposal.

To gain an intuition for why party 1's decision on whether to partner with the high or the low-budget party depends on the type of challenge stage equilibrium considered and on the level of resources, note that the amount of a partner's campaigning resources have two opposing effects on party 1's expected challenge payoff: In an equilibrium with positive contributions from the partner, a higher w_h weakly increases the proposal's winning probability. However, a party also demands a premium over its status-quo payoff from party 1 for agreeing to become an active partner. The analysis indicates that for proposals that imply a challenge with an active partner, the positive effect of a higher w_h on party 1's expected challenge payoff due to a higher probability of winning is dominated by its negative effect due to a higher payment to the partner whenever w_h is too high or party 1's stakes from the challenge are too low. In this case, a higher w_h overall decreases party 1's expected payoff from such a challenge, because the high payment needed to persuade a rich party to become a partner does not justify the increase in party 1's winning probability. On the other hand, for lower values of w_h and $u_1(s)$ that imply high stakes from the challenge, the payment to the partner is justified. In this situation, party 1 would prefer the richer party as its partner.

However, Proposition 4 also indicates that this trade-off between a higher winning probability and a higher partner premium disappears once an equilibrium with a free-rider partner is considered. In these cases, a party can no longer demand a premium for agreeing to become a partner and its budget no longer affects the proposal's probability of winning. However, the opponent's budget w_j negatively affects party 1's expected challenge payoff, giving party 1 the incentive to designate the low-budget party as its opponent.

Given the previous results in Proposition 3 on inducing a grand bargain and the above results on possible challenges, the following proposition presents the main result of this section on party 1's choice between a grand bargain and a challenge outcome:

Proposition 5. *In the political equilibrium of a three-party parliament, party 1 becomes more likely to prefer a grand bargain outcome over a challenge as*

1. *The non-proposer parties command lower status-quo payoffs;*

2. *The three parties get ideologically closer; and*

3. *The non-proposer parties's campaigning budgets become more similar.*

The first and the second parts of Proposition 5 are a direct implication of party 1's unanimity payoff. To see why similar campaigning budgets between the non-proposer parties incentivizes a grand bargain, note that w_2 and w_3 do not affect party 1's unanimity payoff, but determine the proposal needed to induce a given challenge equilibrium. In a challenge stage equilibrium in which the partner also contributes, the premium it demands increases as its resources become more similar to the opponent's, because this increases the competitiveness of the referendum. Since this decreases party 1's expected payoff from this challenge, it will be more likely to prefer a grand bargain.

The results on the proposer's incentives between a grand bargain and a challenge in a three-party parliament mirror those in a two-party parliament. Specifically, the results in these sections indicate that lower status-quo payoffs of the non-proposer parties always incentivize unanimity. Moreover, both sections suggest that a partner's higher budget can be a blessing in a challenge as long as it is not too high, a result that spans both types of parliaments. However, due to the presence of an additional party that the proposer can play against the other, the results on non-cooperative coalition formation are richer in the three-party parliament setting.

3.6 Concluding Remarks

This chapter developed a model of legislative bargaining over a bill consisting of both an ideology and a distributive component followed by a challenge stage. I addressed the question of how an institutional challenge mechanism such as a referendum affects the parties' optimal behavior in a parliament. The analysis of a proposer's incentives between a grand bargain and a challenge indicates that post-bargaining power does not necessarily translate into higher equilibrium payoffs. Although the focus of the model is on legislative bargaining over proposals that can be subsequently challenged, its insights are applicable to other settings, including private sector organizational models. For example, the players in the model can be chosen to represent the board of directors of a corporation, with the chairman as the proposer and shareholders as the voters on proposals not approved with sufficient majority in the board room.

The results of this chapter have implications for campaign finance policies. Even though referenda can be both publicly and privately financed in most countries, this model is silent on this issue. The results for both two and three-party parliaments indicate that whether high or low campaigning budgets incentivize grand bargains depend on the parameters of the model. Therefore, if a planner's goal is to propagate unanimously-approved deals in the parliament over costly challenge procedures, the appropriate campaign finance policy will depend on the status-quo commanded by each party and their current resources.

There exists a number of directions in which the model employed in this chapter can be extended. For example, while I assumed that all the parameters on campaigning budgets, ideal ideological points, and status-quo payoffs are common knowledge, incorporating uncertainty with regards to either one of these parameters can be a natural extension. Although I believe that complete information is a more realistic setting in this model of a public interaction, incomplete information might be a better depiction of reality in private interaction models such as the corporate board example. Extending the model to N players for a more general setting or specifically modeling voters with ideological preferences may also yield interesting results on the dynamics of non-cooperative coalition formation.

Finally, this model does not entertain the possibility of new rounds of bargaining following a challenge stage. However, in reality, political processes might reconsider the same measures. Although I believe that introducing additional cycles of bargaining and challenge stages to this model might make the model much less tractable with little additional insights, it might be a useful endeavor for the purpose of capturing the dynamic aspects of similar political processes. Similarly, an additional stage of legislative elections would make voters strategic by giving them control over the identity of the proposer.

It is important to stress that I do not make any efficiency arguments in favor of one policy over another. For example, if the results suggest caps on campaign financing to incentivize grand bargains for certain ranges of parameters, this study can still not answer the question of how this policy would affect voter welfare. Any attempt to answer this question would require a normative exercise I refrain from.

Appendix A

Maximum Taxes as an Institution

We can alternatively think of an institutional decision as determining the maximum amount of lump-sum taxes, denoted by $\hat{\tau}$, that the next period's incumbent can levy. With this formulation, characterizing the evolution of executive constraints amounts to characterizing the evolution of the maximum level of taxation allowed. In this case, the incumbent in period t would face a single relevant constraint on his policy choice given by

$$\min\left\{\bar{K}, \hat{\tau}_t\right\}, \tag{A.1}$$

where the first term is the exogenous maximum taxable surplus created in the economy and the second term is the executive constraint chosen by the incumbent in period $t-1$. Since it is infeasible to tax above $\bar{K}$, the inequality $\hat{\tau}_t \leq \bar{K}$ always holds so that an incumbent's only constraint becomes $\hat{\tau}_t$.

Since he receives no utility from transferring resources to the other agent, an incumbent i always chooses $y_j = 0$ for $j \neq i$. This implies that we can represent policies

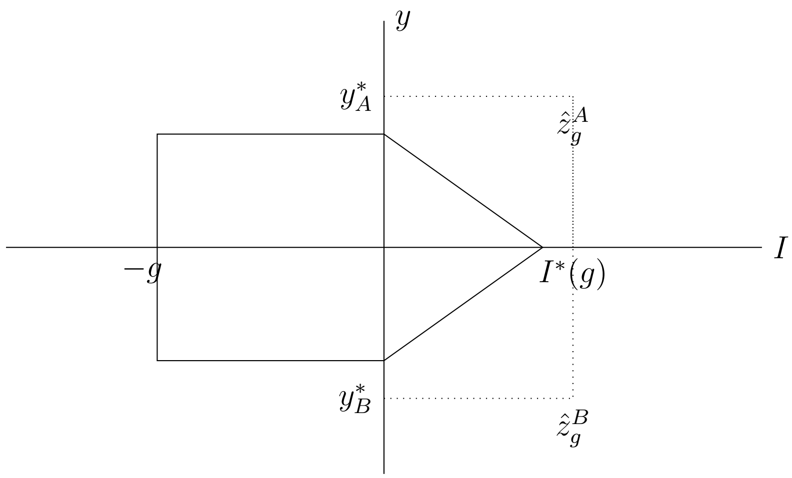

Figure A.1: An example of Γ_t with the institution-as-maximum-taxes approach.

in $\mathbb{R}^2$. To visualize a feasible set in this environment, suppose the stock of public capital is g and the ideal policies for the two agents are represented by $\hat{z}_g^i$. With costless disinvestments, Figure A.1 represents a possible Γ_{t+1} choice for an incumbent.

Although both the model presented here and the one employed in the model treat an institution as a constraint on policy-making, the tax approach is an inadequate substitute for more general institutional constraints. Specifically, while it also imposes simultaneous bounds on private transfer and positive investment decisions, that is not the case with disinvestments. With executive constraints boiling down to a financial constraint, any policy that has a zero price is permitted within this framework. For example, the incumbent is allowed to disinvest the society's entire stock of the public good. In contrast, the structure employed in the model abstracts away from the prices of the policies in its restrictions. Even though disinvestment is a policy with a zero price, it is still restricted as eating up all of a country's public good stock is deemed a too-extreme policy.

Appendix B

Solutions to the Dictatorship Problem

This appendix presents a dictator's optimal choice of investments in periods $t = 1, 2, 3, 4$. Following the notation in the text, let $I_t^*(g_t)$ denote this optimal choice in period t when the level of the public good is given by g_t.

Period 4: Since this is the last period in the model, the optimal decision of a dictator is to not invest in the public good:

$$I_4^*(g_4) = 0. \tag{B.1}$$

Period 3: The dictator chooses $I_3^*(g_3)$ in order to maximize the dynamic component of his utility (since the investment and the private transfer components are separable).

$$\max_{I_3} \bar{A}g_3^{\alpha} - pI_3 + \beta\left[\bar{A}g_4^{\alpha} - pI_4^*(g_4)\right] \tag{B.2}$$

subject to

$$I_3 \geq -g_3; \tag{B.3}$$

$$g_4 = (1-\delta)g_3 + I_3. \tag{B.4}$$

Solving this program yields

$$[(1-\delta)g_3 + I_3]^{\alpha-1} = \frac{p}{\beta\bar{A}\alpha}, \tag{B.5}$$

simplifying which results in

$$I_3^*(g_3) = \left(\frac{\beta\bar{A}\alpha}{p}\right)^{\frac{1}{1-\alpha}} - (1-\delta)g_3. \tag{B.6}$$

Period 2: The dictator solves

$$\max_{I_2} \bar{A}g_2^{\alpha} - pI_2 + \beta\left[\bar{A}g_3^{\alpha} - pI_3^*(g_3)\right] + \beta^2\left[\bar{A}g_4^{\alpha} - pI_4^*(g_4)\right] \tag{B.7}$$

subject to

$$I_2 \geq -g_2; \tag{B.8}$$

$$g_3 = (1-\delta)g_2 + I_2; \tag{B.9}$$

$$g_4 = (1-\delta)g_3 + I_3^*(g_3). \tag{B.10}$$

Solving this program yields

$$\beta \bar{A}\alpha \left[(1-\delta)g_2 + I_2\right]^{\alpha-1} - p\left[1 + \beta\frac{\partial I_3^*(g_3)}{\partial I_2}\right] \tag{B.11}$$

$$+\beta^2 \bar{A}\alpha \left[(1-\delta)^2 g_2 + (1-\delta)I_2 + I_3^*(g_3)\right]^{\alpha-1}\left[(1-\delta) + \frac{\partial I_3^*(g_3)}{\partial I_2}\right] = 0.$$

Since B.6 implies

$$\frac{\partial I_3^*(g_3)}{\partial I_2} = -(1-\delta), \tag{B.12}$$

the final component of B.11 disappears, resulting in

$$\beta \bar{A}\alpha \left[(1-\delta)g_2 + I_2\right]^{\alpha-1} - p[1-\beta+\beta\delta] = 0. \tag{B.13}$$

Solving B.13 for I_2 yields

$$I_2^*(g_2) = \left(\frac{\beta\bar{A}\alpha}{p(1-\beta+\beta\delta)}\right)^{\frac{1}{1-\alpha}} - (1-\delta)g_2. \tag{B.14}$$

Period 1: In the first period, the dictator solves

$$\max_{I_1}\ \bar{A}g_1^\alpha - pI_1 + \sum_{t=2}^{4}\beta^{t-1}[\bar{A}g_t^\alpha - pI_t^*(g_t)] \tag{B.15}$$

subject to

$$I_1 \geq -g_1; \tag{B.16}$$

$$g_2 = (1-\delta)g_1 + I_1; \tag{B.17}$$

$$g_3 = (1-\delta)g_2 + I_2^*(g_2); \tag{B.18}$$

$$g_4 = (1-\delta)g_3 + I_3^*(g_3). \tag{B.19}$$

Solving this program yields

$$\beta\bar{A}\alpha\left[(1-\delta)g_1 + I_1\right]^{\alpha-1} - p\left[1 + \beta\frac{\partial I_2^*(g_2)}{\partial I_1} + \beta^2\frac{\partial I_3^*(g_3)}{\partial I_1}\right] \tag{B.20}$$

$$+\beta^2\bar{A}\alpha\left[(1-\delta)^2 g_1 + (1-\delta)I_1 + I_2^*(g_2)\right]^{\alpha-1}\left[(1-\delta) + \frac{\partial I_2^*(g_2)}{\partial I_1}\right]$$

$$+\beta^3\bar{A}\alpha\left[(1-\delta)^3 g_1 + (1-\delta)^2 I_1 + (1-\delta)I_2^*(g_2) + I_3^*(g_3)\right]^{\alpha-1}\left[(1-\delta)^2 + (1-\delta)\frac{\partial I_2^*(g_2)}{\partial I_1} + \frac{\partial I_3^*(g_2)}{\partial I_1}\right]$$

$$= 0.$$

Since B.14 implies

$$\frac{\partial I_2^*(g_2)}{\partial I_1} = -(1-\delta), \tag{B.21}$$

the second line in B.20 disappears. Moreover, B.6 implies

$$\frac{\partial I_3^*(g_2)}{\partial I_1} = 0. \tag{B.22}$$

Thus, we can re-write B.20 as follows:

$$\beta\bar{A}\alpha\left[(1-\delta)g_1 + I_1\right]^{\alpha-1} = p[1-\beta+\beta\delta]. \tag{B.23}$$

Solving B.23 for I_1 yields

$$I_1^*(g_1) = \left(\frac{\beta \bar{A} \alpha}{p(1-\beta+\beta\delta)}\right)^{\frac{1}{1-\alpha}} - (1-\delta)g_1. \tag{B.24}$$

The equation B.24 expresses the dictator's optimal choice of investment in period 1 only as a function of the parameters of the model and the exogenously given g_1. Using the fact that $g_2 = (1-\delta)g_1 + I_1^*(g_1)$ when the dictator is behaving optimally and substituting B.24 into B.14, we get

$$I_2^*(g_2) = \delta \left(\frac{\beta \bar{A} \alpha}{p(1-\beta+\beta\delta)}\right)^{\frac{1}{1-\alpha}}. \tag{B.25}$$

Similarly, substituting B.14 and B.24 into B.6 yields

$$I_3^*(g_3) = \left(\frac{\beta \bar{A} \alpha}{p}\right)^{\frac{1}{1-\alpha}} \left[1 - \frac{(1-\delta)^2}{(1-\beta+\beta\delta)^{\frac{1}{1-\alpha}}} - \frac{\delta(1-\delta)}{(1-\beta+\beta\delta)^{\frac{1}{1-\alpha}}}\right]. \tag{B.26}$$

The solutions in this Appendix will provide the basis for evaluating public good provision under political uncertainty.

Appendix C

Proofs for Chapter 1

Proof of Lemma 1. Based on the definition of an institutionally feasible set $\Gamma(r)$ as a circle with origin $(0,0)$ and radius r, defining $\bar{r}(g) = \min\{r \mid (I^*(g), y_i^*) \in \Gamma(r)\}$ for $i = A, B$ implies that

$$\bar{r}(g) = \sqrt{(I^*(g))^2 + (y_i^*)^2}. \tag{C.1}$$

Since the amount of ideal private transfers to an incumbent is the same regardless of his identity, I omit the subscript i from y_i^* in the following analysis.

First, consider $g > \hat{g}$ so that $\gamma^*(g) = (1-\delta)\hat{g}$ and $I^*(g) = (1-\delta)\hat{g} - (1-\delta)g = -(1-\delta)(g-\hat{g})$. Let g_k for $k = 1, 2, 3$ denote a state of the public good such that $g_k > \hat{g}$ for all k, $g_1 > g_2 > g_3$, and $\mid g_1 - g_2 \mid = \mid g_2 - g_3 \mid$. Define the function $I^* : \mathbb{R}_+ \to \mathbb{R}$, where $I^*(g)$ is the (common) ideal level of investment when the public good is given by g. Note that the values of the function I^* are equivalent to the levels of investment that would be chosen by either dictator type for any given level of g. Since I^* is a linear function for all $g > \hat{g}$ with a first derivative equal to $-(1-\delta)$, it follows that $| I^*(g_1) - I^*(g_2) |=| I^*(g_2) - I^*(g_3) |$. By the equivalence of either dictator type's transfers to himself and the fact that $| I^*(g_1) |>| I^*(g_2) |>| I^*(g_3) |$, it follows that

$$\bar{r}(g_1) > \bar{r}(g_2) > \bar{r}(g_3). \tag{C.2}$$

Hence, $\bar{r}$ is increasing for all $g > \hat{g}$.

To see why $\bar{r}$ is convex for $g > \hat{g}$, consider the second derivative of $\bar{r}(g) = \sqrt{(y^*)^2 + [(1-\delta)(\hat{g}-g)]^2}$. Differentiating $\bar{r}(g)$ with respect to g yields

$$\frac{d\bar{r}(g)}{dg} = \left[(y^*)^2 + (1-\delta)^2(\hat{g}-g)^2\right]^{-\frac{1}{2}} (1-\delta)^2 (g-\hat{g}), \tag{C.3}$$

which confirms that $\bar{r}$ is increasing since $g > \hat{g}$. Differentiate C.3 with respect to g again to obtain

$$-(1-\delta)^4 \left[(y^*)^2 + (1-\delta)^2(\hat{g}-g)^2\right]^{-\frac{3}{2}} (\hat{g}-g)^2 + (1-\delta)^2 \left[(y^*)^2 + (1-\delta)^2(\hat{g}-g)^2\right]^{-\frac{1}{2}},$$

which yields

$$[1-\delta]^2 \left[(y^*)^2 + (1-\delta)^2(\hat{g}-g)^2\right]^{-\frac{1}{2}} \left[1 - \frac{(1-\delta)^2(g-\hat{g})^2}{[(y^*)^2 + (1-\delta)^2(\hat{g}-g)^2]}\right]. \tag{C.4}$$

Since $(y^*)^2 > 0$, the value of the fraction in C.4 is less than 1 and positive. Hence, the final term in brackets in C.4 is positive so that the second derivative of the function $\bar{r}$ becomes positive for all $g > \hat{g}$.

Second, consider $g < \hat{g}$, where the dictatorial investment rule γ^* is increasing and concave. Accordingly, for any g_k for $k = 1, 2, 3$ such that $g_k < \hat{g}$ for all k, $g_1 < g_2 < g_3$, and $g_2 - g_1 = g_3 - g_2$, the equality $I^*(g_1) - I^*(g_2) = I^*(g_3) - I^*(g_2)$ no longer holds since the function I^* is no longer linear. In contrast, $I^*(g)$ decreases at a decreasing rate, albeit taking only positive values since we are below the investment cut-off state. In this case, differentiating $\bar{r}(g)$ with respect to g yields

$$\frac{d\bar{r}(g)}{dg} = \left[(y^*)^2 + (I^*(g))^2\right]^{-\frac{1}{2}} \frac{dI^*(g)}{dg}. \tag{C.5}$$

Since $I^*(g)$ is decreasing for all $g < \hat{g}$, it follows that C.5 is negative. To obtain the second-order properties of $\bar{r}$ below the investment cut-off state, differentiate C.5 once more to get

$$\frac{d^2\bar{r}(g)}{dg^2} = \left[(y^*)^2 + (I^*(g))^2\right]^{-\frac{1}{2}} \frac{d^2I^*(g)}{dg^2} - \left[(y^*)^2 + (I^*(g))^2\right]^{-\frac{3}{2}} \left(\frac{dI^*(g)}{dg}\right)^2. \tag{C.6}$$

Since the second derivative of I^* with respect to g is negative for all $g > \hat{g}$, C.6 implies that the second derivative of the $\bar{r}$ function with respect to g is negative for all $g < \hat{g}$.

Finally, suppose $g = \hat{g}$ so that $\gamma^*(\hat{g}) = (1-\delta)\hat{g}$, which implies $I^*(\hat{g}) = 0$. By the continuity of I^* and the fact that y^* is constant for both agents, $\bar{r}$ is continuous. Since $\frac{d\bar{r}(g)}{dg} < 0$ for all $g < \hat{g}$ and $\frac{d\bar{r}(g)}{dg} > 0$ for all $g > \hat{g}$, it follows that the derivative of the function $\bar{r}$ evaluated at $g = \hat{g}$ is equal to 0 and $\hat{g} \in \underset{g}{argmin}\ \bar{r}(g)$. Moreover, the argmin set is a singleton. Hence, this completes the proof that the function $\bar{r}$ is increasing at an increasing rate as g moves away from the investment cut-off state $\hat{g}$. $\square$

Proof of Proposition 1. The solution to the second period incumbent's problem is summarized in the text. Here, I only solve for optimal behavior in the first period. By the same argument that led to 1.23, we have $\Upsilon^i_{j,1}(r_1, g_1) = 0$ for all r_1 and g_1, and $j \neq i$. To solve for the optimal choices of I_1, $y_{i,1}$, and r_2, define the Lagrangian for $\kappa_1 = i$ as follows:

$$L_i = y_{i,1} - x(y_{i,1}) - x(y_{j,1}) + \bar{A}g_1^\alpha - pI_1 \tag{C.7}$$

$$+\beta[q_i V_2^i(r_2, g_2) + (1 - q_i)W_2^i(r_2, g_2)]$$

$$+\lambda_1[(r_1)^2 - (I_1)^2 - (y_{i,1})^2] + \lambda_2(I_1 + g_1),$$

where $j \neq i$. The first-order conditions for C.7 are $y_{i,1} \geq 0$, $I_1 \geq -g_1$, $r_2 \geq 0$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and

$$1 - b(y_{i,1})^{b-1} - 2\lambda_1 y_{i,1} \leq 0; \tag{C.8}$$

$$[1 - b(y_{i,1})^{b-1} - 2\lambda_1 y_{i,1}]\ y_{i,1} = 0; \tag{C.9}$$

$$\bar{A}\beta\alpha[(1-\delta)g_1 + I_1]^{\alpha-1} - p - 2\lambda_1 I_1 + \lambda_2 \leq 0; \tag{C.10}$$

$$[\bar{A}\beta\alpha[(1-\delta)g_1 + I_1]^{\alpha-1} - p - 2\lambda_1 I_1 + \lambda_2]\ [I_1 + g_1] = 0; \tag{C.11}$$

$$\beta q_i \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2}[1 - b(\Upsilon^i_{i,2}(r_2, g_2))^{b-1}] - \beta q_j \frac{\partial \Upsilon^j_{j,2}(r_2, g_2)}{\partial r_2}[b(\Upsilon^j_{j,2}(r_2, g_2))^{b-1}] \leq 0; \tag{C.12}$$

$$\left[\beta q_i \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2}[1 - b(\Upsilon^i_{i,2}(r_2, g_2))^{b-1}] - \beta q_j \frac{\partial \Upsilon^j_{j,2}(r_2, g_2)}{\partial r_2}[b(\Upsilon^j_{j,2}(r_2, g_2))^{b-1}]\right]\ r_2 = 0; \tag{C.13}$$

$$(r_1)^2 - (I_1)^2 - (y_{i,1})^2 \geq 0; \tag{C.14}$$

$$[(r_1)^2 - (I_1)^2 - (y_{i,1})^2]\ \lambda_1 = 0; \tag{C.15}$$

$$I_1 + g_1 \geq 0; \tag{C.16}$$

$$(I_1 + g_1)\ \lambda_2 = 0. \tag{C.17}$$

Note that C.12 and C.13 already reflect the fact that $\Upsilon^i_{j,2}(r_2, g_2) = 0$ for $j \neq i$.

Consider the policy choices of incumbent κ_1. First, I look for a solution where $\lambda_1 = 0$ and $\lambda_2 = 0$ so that neither of the two constraints is binding. When this is the case, κ_1 chooses his ideal level of investment and private transfers to himself so that

$$\gamma^i_1(r_1, g_1) = \left(\frac{\bar{A}\beta\alpha}{p}\right)^{\frac{1}{1-\alpha}} \tag{C.18}$$

and

$$\Upsilon^i_{i,1}(r_1, g_1) = b^{\frac{1}{1-b}}. \tag{C.19}$$

Second, I look for a solution where $\lambda_1 = 0$, so that the executive constraint is not binding, and $\lambda_2 > 0$. This implies that $I_1 = -g_1$ and therefore $\gamma^i_1(r_1, g_1) = 0$. Since r_1 is not binding, incumbent i chooses $y_{i,1}$ according to C.19. This solution implies that $\lambda_2 = p$.

Finally, I look for a solution where $\lambda_1 > 0$ and $\lambda_2 = 0$. Then, C.9 and C.11 imply that

$$1 - b(y_{i,1})^{b-1} - 2\lambda_1 y_{i,1} = 0 \tag{C.20}$$

and

$$\bar{A}\beta\alpha[(1-\delta)g_1 + I_1]^{\alpha-1} - p - 2\lambda_1 I_1 = 0. \tag{C.21}$$

Moreover, $\lambda_1 > 0$ implies that the executive constraint is binding and hence $y_{i,1} = \sqrt{(r_1)^2 - (I_1)^2}$. Substituting this equality for $y_{i,1}$ into C.21 yields two equations with two unknowns, λ_1 and I_1. Solving this system of equations yields the implicit equation 1.26 that defines the optimal choice of investment in period 1 when r_1 is binding. Accordingly, we characterize the optimal choice of private transfers for agent i himself by 1.29.

Now, consider the institutional choice of incumbent κ_1. An interior solution to the optimal level of r_2 implies that

$$\beta q_i \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2}[1 - b(\Upsilon^i_{i,2}(r_2, g_2))^{b-1}] - \beta q_j \frac{\partial \Upsilon^j_{j,2}(r_2, g_2)}{\partial r_2}[b(\Upsilon^j_{j,2}(r_2, g_2))^{b-1}] = 0. \tag{C.22}$$

Notice that regardless of an incumbent's identity, the optimal *amount* of private transfers is always the same to the incumbent himself. Therefore, $\Upsilon^i_{i,t}(r_t, g_t) = \Upsilon^j_{j,t}(r_t, g_t) \ \forall \ (r_t, g_t) \in \mathbb{R}^2_+$ and t. Simplifying C.22 using this identity yields

$$q_i \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2} - b(\Upsilon^i_{i,2}(r_2, g_2))^{b-1} \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2} = 0, \tag{C.23}$$

which implies

$$[q_i - b(\Upsilon^i_{i,2}(r_2, g_2))^{b-1}] \frac{\partial \Upsilon^i_{i,2}(r_2, g_2)}{\partial r_2} = 0. \tag{C.24}$$

This is because an agent i always pays for a unit of private transfers through taxes regardless of whether he is the recipient or not, but only enjoys it if he is receiving the transfer.

By 1.24, $\Upsilon^i_{i,2}(r_2, g_2) = r_2$ whenever r_2 is binding. Hence, C.24 becomes

$$q_i - b(r_2)^{b-1} = 0. \tag{C.25}$$

This implies that the equilibrium institutional strategy of $\kappa_1 = i$ is given by

$$\rho^i_1(r_1, g_1) = \left(\frac{b}{q_i}\right)^{\frac{1}{1-b}}. \tag{C.26}$$

On the other hand, if $\Upsilon^i_{i,2}(r_2, g_2) = b^{\frac{1}{1-b}}$, any r_2 such that $r_2 \geq b^{\frac{1}{1-b}}$ is optimal. Hence, any institutional strategy ρ^i_1 such that $\rho^i_1(r_1, g_1) \geq b^{\frac{1}{1-b}}$ constitutes an equilibrium strategy.

This completes the full characterization of an incumbent's policy and institutional strategies when $T = 2$. $\square$

Proof of Proposition 2. I solve the four-period model via backward induction. First, consider the private transfer strategies of incumbent κ_t as described in Part 1 of the proposition. Since incumbent i does not receive any utility from making positive transfers to the other agent, we have $\Upsilon^i_{j,t}(r_t, g_t) = 0$ for all t and $j \neq i$. For private transfers to himself, incumbent i's optimal decision is given by C.19 from the proof of Proposition 1 when r_t is not binding, and by $\Upsilon^i_{i,t}(r_t, g_t) = \sqrt{(r_t)^2 - (I_t)^2}$ when it is, where $I_t = \gamma^i_t(r_t, g_t) - (1-\delta)g_t$. Since the private transfer decision is purely static, this is true for all t.

Second, consider the investment decision of incumbent i for all periods, described in Part 2 of the proposition.

Since period-4 is the final period, the optimal decision is to not invest so that the optimal I_4 equals zero and $\gamma_4^i(r_4, g_4) = (1-\delta)g_4$.

Since period-3 when $T = 4$ is equivalent to the first period in the two-period model considered in the previous section, the optimal I_3 is obtained by solving C.7, which yields the first-order conditions C.10, C.11, C.15 and C.17. As summarized in Proposition 1, this yields 1.26 and 1.27 for the optimal choice of I_3. Note that there exists no part in either expression that is agent-specific. Therefore, the optimal choice of I_3 is the same for both types of agents.

For period-2, the Lagrangian for incumbent κ_2 can be written as

$$L_i^2 = y_{i,2} - x(y_{i,2}) - x(y_{j,2}) + \bar{A}g_2^{\alpha} - pI_2 \tag{C.27}$$

$$+\beta[q_i V_3^i(r_3, g_3) + (1-q_i)W_3^i(r_3, g_3)]$$

$$+q_i\beta^2[q_i V_4^i(\rho_3^i(r_3, g_3), \gamma_3^i(r_3, g_3)) + (1-q_i)W_4^i(\rho_3^i(r_3, g_3), \gamma_3^i(r_3, g_3))]$$

$$+q_j\beta^2[q_i V_4^i(\rho_3^j(r_3, g_3), \gamma_3^j(r_3, g_3)) + (1-q_i)W_4^i(\rho_3^j(r_3, g_3), \gamma_3^j(r_3, g_3))]$$

$$+\lambda_1[(r_2)^2 - (I_2)^2 - (y_{i,2}^2)] + \lambda_2[I_2 + g_2],$$

where $j \neq i$. We have already shown in Part 1 of the proposition that $\Upsilon_{j,t}^i(r_t, g_t) = 0$ for all t so that the optimal choice of $y_{j,2}$ is zero. Using the fact that $\gamma_3^A(r_3, g_3) = \gamma_3^B(r_3, g_3)$ for all r_3 and g_3, the first-order conditions based on C.27 for incumbent i's policy decisions are $y_{i,2} \geq 0$, $I_2 \geq -g_2$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and

$$1 - b(y_{i,2})^{b-1} - 2\lambda_1 y_{i,2} \leq 0; \tag{C.28}$$

$$\left[1 - b(y_{i,2})^{b-1} - 2\lambda_1 y_{i,2}\right] \, y_{i,2} = 0; \tag{C.29}$$

$$\bar{A}\beta\alpha[(1-\delta)g_2 + I_2]^{\alpha-1} - p - \beta p \frac{\partial I_3}{\partial g_3} \tag{C.30}$$

$$+q_i\beta^2 \sum_{k=A,B} q_k \frac{\partial \Upsilon^i_{i,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3))}{\partial r_4} \frac{\partial \rho^k_3(r_3,g_3)}{\partial g_3} \left[1 - b(\Upsilon^i_{i,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3)))^{b-1}\right]$$

$$-q_j\beta^2 \sum_{k=A,B} q_k \frac{\partial \Upsilon^j_{j,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3))}{\partial r_4} \frac{\partial \rho^k_3(r_3,g_3)}{\partial g_3} b(\Upsilon^j_{j,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3)))^{b-1}$$

$$-2\lambda_1 I_2 + \lambda_2 \leq 0;$$

$$[\bar{A}\beta\alpha[(1-\delta)g_2 + I_2]^{\alpha-1} - p - \beta p \frac{\partial I_3}{\partial g_3} \tag{C.31}$$

$$+q_i\beta^2 \sum_{k=A,B} q_k \frac{\partial \Upsilon^i_{i,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3))}{\partial r_4} \frac{\partial \rho^k_3(r_3,g_3)}{\partial g_3} \left[1 - b(\Upsilon^i_{i,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3)))^{b-1}\right]$$

$$-q_j\beta^2 \sum_{k=A,B} q_k \frac{\partial \Upsilon^j_{j,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3))}{\partial r_4} \frac{\partial \rho^k_3(r_3,g_3)}{\partial g_3} b(\Upsilon^j_{j,4}(\rho^k_3(r_3,g_3), \gamma^k_3(r_3,g_3)))^{b-1}$$

$$-2\lambda_1 I_2 + \lambda_2][I_2 + g_2] = 0;$$

$$(r_2)^2 - (I_2)^2 - (y_{i,2})^2 \geq 0; \tag{C.32}$$

$$\left[(r_2)^2 - (I_2)^2 - (y_{i,2})^2\right] \, \lambda_1 = 0; \tag{C.33}$$

$$I_2 + g_2 \geq 0; \tag{C.34}$$

$$(I_2 + g_2) \; \lambda_2 = 0. \tag{C.35}$$

I first look for a solution in which $\lambda_1 > 0$ and $\lambda_2 = 0$. Solving for the optimal I_2 yields the implicit equation in 1.31 when r_2 is binding. Since $\gamma_3^A(r_3, g_3) = \gamma_3^B(r_3, g_3)$ for all $(r_3, g_3) \in \mathbb{R}_+^2$ and 1.31 does not contain agent-specific parameters, it follows that the agents follow the same γ_2^i rule.

Solving for the optimal first period investment choice of incumbent κ_1 requires writing a period-1 Lagrangian as in C.27 that accounts for three-period forward-looking. This yields the implicit equation 1.32. The analysis is similar to the analysis for period-2 and therefore will not be repeated here. This completes the proof of Part 2 of the proposition.

Finally, consider the institutional strategies of incumbent κ_t in all periods.

The optimal choice of r_4 in period-3 is equivalent to the only institutional decision in the two-period model and hence is characterized by 1.30.

Consider the r_3 decision of incumbent κ_2. Based on C.27, the first-order conditions for the institutional choice in period-3 can be written as

$$r_3 \geq 0; \tag{C.36}$$

$$-\beta \left[p - \bar{A}\alpha g_4^{\alpha-1}\right] \frac{\partial I_3}{\partial r_3} + \left[q_i - b(\Upsilon_{i,3}^i(r_3, g_3))^{b-1}\right] \frac{\partial \Upsilon_{i,3}^i(r_3, g_3)}{\partial r_3} \leq 0; \tag{C.37}$$

and

$$\left[-\beta[p - \bar{A}\alpha g_4^{\alpha-1}]\frac{\partial I_3}{\partial r_3} + \left[q_i - b(\Upsilon_{i,3}^i(r_3, g_3))^{b-1}\right] \frac{\partial \Upsilon_{i,3}^i(r_3, g_3)}{\partial r_3}\right] r_3 = 0. \tag{C.38}$$

Note that the period-4 policies drop from the first-order conditions for the following two reasons: First, the optimal r_4 as defined in 1.30 does not depend on r_3. Second, possible final period policies given by $\Upsilon^k_{i,4}(r_4, g_4)$ and $\gamma^k_4(r_4, g_4)$ for $k = A, B$ do not depend on g_4, because the optimal investment I_4 is always zero and as a result the optimal private transfers only depend on r_4, which does not depend on r_3. Hence, an interior solution to r_3 is implicitly defined by

$$\left[q_i - b(\Upsilon^i_{i,3}(r_3, g_3))^{b-1}\right] \frac{\partial \Upsilon^i_{i,3}(r_3, g_3)}{\partial r_3} = \mid p - \bar{A}\alpha g_4^{\alpha-1} \mid\mid \frac{\partial I_3}{\partial r_3} \mid, \tag{C.39}$$

where $I_3 = \gamma^i_3(r_3, g_3) - (1-\delta)g_3$ and the absolute values are necessary to account for disinvestments.

Finally, solving for the optimal r_2 decision of incumbent κ_1 again requires writing a period-1 Lagrangian that accounts for three-period forward-looking. This analysis yields the implicit equation 1.34. This completes the proof of Proposition 2. □

Proof of Proposition 3. For $t = 3$, the conclusion of the proposition is straightforward: As q_i increases, the value of ρ^i_3 increases for any $g_3 \in \mathbb{R}_+$ since $b > 1$.

To show that the optimal choice of r_3 is increasing with q_i, I implicitly differentiate equation 1.33 to get the following expression for $\frac{\partial \rho^i_2(r_2,g_2)}{\partial q_i}$: $\frac{\partial \Upsilon^i_{i,3}(r_3,g_3)}{\partial r_3}$ divided by

$$\bar{A}(1-\alpha)\alpha g_4^{\alpha-2}\frac{\partial g_4}{\partial r_3} + \bar{A}\alpha g_4^{\alpha-1} \mid \frac{\partial^2 I_3}{\partial (r_3)^2} \mid \tag{C.40}$$

$$+b(b-1)\Upsilon^i_{i,3}(r_3, g_3)^{b-2}\left(\frac{\partial \Upsilon^i_{i,3}(r_3, g_3)}{\partial r_3}\right)^2 + \left[b\Upsilon^i_{i,3}(r_3, g_3)^{b-1} - q_i\right]\frac{\partial^2 \Upsilon^i_{i,3}(r_3, g_3)}{\partial (r_3)^2}.$$

Note that the first, second, and the third expressions in C.40 are always non-negative. In the fourth component of the summation, $b\Upsilon^i_{i,3}(r_3, g_3)^{b-1} - q_i$ is always non-positive, because an incumbent would never choose a level of private transfers at which the marginal cost of an additional unit, $b\Upsilon^i_{i,3}(r_3, g_3)^{b-1}$ exceeds its expected marginal benefit, q_i. Since $\Upsilon^i_{i,3}(r_3, g_3)$ equals $\sqrt{(r_3)^2 - (I_3)^2}$ when r_3 is binding, its second derivative is non-positive. Hence, the fourth component of C.40 is non-negative as well. Therefore, C.40 is non-negative, i.e.

$$\frac{\partial \rho^i_2(r_2, g_2)}{\partial q_i} \geq 0. \tag{C.41}$$

The second-period analysis is identical and hence is not repeated here. Therefore, we conclude that re-election probability and the level of executive constraints move in the same direction. □

Proof of Proposition 4. Implicitly differentiating 1.33 with respect to g_2 and evaluating the resulting expression on both sides of $\hat{g}$ yield the desired result that $\frac{\partial r_3}{\partial g_2}$ is negative for all $g < \hat{g}$ and positive for all $g > \hat{g}$.

A more intuitive proof is obtained by noting that as g approaches $\hat{g}$ from either direction, the expression $\mid p - \bar{A}\alpha g_4^{\alpha-1} \mid$ goes to zero. This implies setting r_3 with more weight on private transfer preferences and less weight on investment. For example, when $g = \hat{g}$, r_3 is set such that $\Upsilon^i_{i,3}(r_3, g_3) = \left(\frac{b}{q_i}\right)^{\frac{1}{1-b}}$. As g moves away from $\hat{g}$, resulting in investment or disinvestment needs, $r_3 = \sqrt{(y_3)^2 + (I_3)^2}$ must increase to accommodate them. Therefore, r_3 must be increasing as g_2 moves away from $\hat{g}$ in either direction. This concludes the proof of the first part of Proposition 4.

To see that the difference between $\bar{r}(g_t)$ and $\rho^i_t(r_t, g_t)$ is positive for all possible values of r_t and g_t, recall from Lemma 1 that $\bar{r}(g) = \sqrt{(I^*(g_t))^2 + (y_i^*)^2}$. Hence,

$$\sqrt{(I^*(g_t))^2 + (y_i^*)^2} - \sqrt{[\gamma_t^i(r_t, g_t) - (1-\delta)g_t]^2 + [\Upsilon_{i,t}^i(r_t, g_t)]^2} \tag{C.42}$$

must be always positive since $\mid I^*(g_t) \mid > \mid \gamma_t^i(r_t, g_t) - (1-\delta)g_t \mid$ and $y_i^* > \Upsilon_{i,t}^i(r_t, g_t)$ for all $g_t \in \mathbb{R}_+$ and $r_t > 0$. □

Proof of Proposition 5. The equation 1.32 that implicitly defines the optimal choice of I_1 contains a single agent-specific parameter, which is q_i. Hence, as the difference between q_A and q_B increases, the difference between the agents' optimal choices of I_1 will increase. □

Proof of Proposition 6. Since the optimal choice of I_4 always equals zero, I begin the comparison of political equilibrium with the dictatorial benchmarks established in Appendix B in period 3. In that period, the dictator invests in the amount of $I_3^*(g_3)$ as characterized in B.6, whereas incumbent i facing political uncertainty chooses $I_3 = \gamma_3^i(r_3, g_3) - (1-\delta)g_3$ as given in 1.26 and 1.27. Note that B.6 and 1.27 indicate the same choices of investment for the dictator and incumbent i, because 1.27 corresponds to the case in which r_3 is not binding. On the other hand, if the executive constraint r_3 is binding so that the political I_3 choice is as given in 1.26, we have $I_3 < I_3^*(g_3)$. To see this, note that 1.26 is derived using the first-order condition

$$\bar{A}\beta\alpha[(1-\delta)g_3 + I_3]^{\alpha-1} - 2\lambda_1 I_3 = p, \tag{C.43}$$

where $\lambda_1 > 0$. Comparing C.43 with B.5 indicates that the marginal benefit of investing under political uncertainty is reduced by the value of $2\lambda_1 I_3$, which implies a lower optimal choice of I_3 compared to the dictator's.

For $t = 2$, compare the dictatorial decision $I_2^*(g_2)$ given in B.14 with the political choice of I_2 implicitly defined in 1.31. By the same argument as in the previous paragraph and the added effect of $\beta p \frac{\partial I_3}{\partial g_3}$ that contributes to a lower marginal benefit of investing, we have $I_2 < I_2^*(g_2)$. The fourth period follows the same argument. Hence, this completes the proof that for all t, $I_t^*(g_t) \geq I_t$ for $g < \hat{g}$ and $I_t^*(g_t) \leq I_t$ for $g > \hat{g}$. $\square$

Appendix D

Proofs for Chapter 2

Proof of Proposition 1. Let $T = 3$.[1] Since there exist no institutional or investment decisions in the final period, consider $t = 2$. Note that investing in period 2 determines the level of c_3, which in turn determines the cost of choosing $\ell_4 \neq \ell_3$. Because there exists no ℓ_4 decision, the dictator optimally chooses $I_2 = 0$ so that $\gamma_2^*(\ell_2, c_2) = (1-\delta)c_2$.

Given that he neither invests nor disinvests in c_2, the dictator's payoff in $t = 2$ from choosing $\ell_3 \neq \ell_2$ is given by

$$-d(\hat{p}_A, p_2^*(\ell_2)) - c_2 - \beta d(\hat{p}_A, \hat{p}_A), \tag{D.1}$$

whereas his payoff from keeping $\ell_3 = \ell_2$ is

$$-(1+\beta)d(\hat{p}_A, p_2^*(\ell_2)). \tag{D.2}$$

[1] This is also the number of periods for which the model under political uncertainty will be solved.

The dictator reforms the executive constraints whenever his dynamic payoff from doing so is greater than his payoff from maintaining the status-quo. Hence, based on D.1 and D.2, it is profitable for the dictator to choose $\ell_3 \neq \ell_2$ whenever

$$c_2 \leq \beta d(\hat{p}_A, p_2^*(\ell_2)), \tag{D.3}$$

since $d(\hat{p}_A, \hat{p}_A) = 0$, $p_3^*(\ell_3) = p_2^*(\ell_2)$ if $\ell_3 = \ell_2$, and $\ell_3 \neq \ell_2$ is such that $\hat{p}_A \in \Gamma(\ell_3)$. Condition D.3 indicates that dictator A reforms the executive constraints in $t = 2$ if the policy cost of keeping $\ell_3 = \ell_2$ justifies paying the cost of reform given by c_2.

Now consider $t = 1$. Given ℓ_1 and the optimal I_1 decision that will be subsequently analyzed, the dictator's payoff from choosing $\ell_2 \neq \ell_1$ is

$$-d(\hat{p}_A, p_1^*(\ell_1)) - c_1 - I_1 - \sum_{t=2}^{3} \beta^{t-1} d(\hat{p}_A, \hat{p}_A), \tag{D.4}$$

since choosing $\ell_2 \neq \ell_1$ such that $\hat{p}_A \in \Gamma(\ell_2)$ allows the dictator to have $p_t^*(\ell_t) = \hat{p}_A$ for $t = 2$ and 3. On the other hand, if he maintains the status-quo, then his dynamic payoff depends on whether he will choose $\ell_3 = \ell_2$ in $t = 2$ or not. Specifically, if condition D.3 holds, which can be re-written as

$$(1 - \delta)c_1 + I_1 \leq \beta d(\hat{p}_A, p_2^*(\ell_2)), \tag{D.5}$$

the dictator will reform the executive constraints in $t = 2$ and his payoff from choosing $\ell_2 = \ell_1$ in $t = 1$ becomes

$$-d(\hat{p}_A, p_1^*(\ell_1)) - I_1 - \beta d(\hat{p}_A, p_2^*(\ell_1)) - \beta c_2 - \beta^2 d(\hat{p}_A, \hat{p}_A). \tag{D.6}$$

In contrast, if D.3 does not hold so that the dictator will not reform in $t = 2$, his payoff from keeping $\ell_2 = \ell_1$ is given by

$$-d(\hat{p}_A, p_1^*(\ell_1)) - I_1 - \beta d(\hat{p}_A, p_2^*(\ell_1)) - \beta^2 d(\hat{p}_A, p_3^*(\ell_1)). \tag{D.7}$$

When D.5 holds, comparing D.4 and D.6 yields the condition that needs to hold for the dictator to reform in $t = 1$ when he would reform in $t = 2$ if he did not reform today. Likewise, when D.5 does not hold, comparing D.4 and D.7 yields the similar condition for the case in which the dictator would not reform thereafter if he did not reform today. Note that D.5 cannot hold if the dictator has already reformed in $t = 1$ so that $\hat{p}_A \in \Gamma(\ell_2)$. This is because $\ell_2 \neq \ell_1$ would imply $p_2^*(\ell_2) = \hat{p}_A$, thereby making $d(\hat{p}_A, p_2^*(\ell_2)) = 0$. Since it is not possible to have $c_2 \leq 0$, we must have $\ell_2 = \ell_1$ whenever D.5 holds.

Since these comparisons depend on the optimal I_1 in each scenario, we first need to characterize its choice in equilibrium. First, notice that the dictator will never choose $I_1 > 0$ so that $\gamma_1^*(\ell_1, c_1) \in [0, (1-\delta)c_1]$. Second, if dictator A chooses $\ell_2 \neq \ell_1$, resulting in the payoff given in D.4, he will let $I_1 = 0$.

For the optimal investment choice in the two scenarios whose payoffs are represented by D.6 and D.7, first suppose the dictator chooses I_1 and ℓ_2 such that D.5 holds, i.e. that the resulting c_2 would allow him to reform the constraints in $t = 2$ if he did not already reform in $t = 1$. Since D.5 can only hold if $\ell_1 = \ell_2$, I_1 has to be such that

$$I_1 \leq \min\{0, d(\hat{p}_A, p_1^*(\ell_1)) - (1-\delta)c_1\}. \tag{D.8}$$

In this case, dictator A's payoff from choosing $\ell_2 = \ell_1$ and $\ell_3 \neq \ell_2$ becomes

$$-d(\hat{p}_A, p_1^*(\ell_1)) - \mid I_1 \mid - \beta d(\hat{p}_A, p_2^*(\ell_1)) - \beta[(1-\delta)c_1 + I_1] - \beta^2 d(\hat{p}_A, \hat{p}_A), \tag{D.9}$$

where the last term equals zero. Choosing I_1 in order to maximize D.9 implies that since the dictator always pays $\mid I_1 \mid$ for any disinvestment today just to recoup it tomorrow in cost savings of $\beta \mid I_1 \mid$, any I_1 that satisfies D.8 is a solution to this problem. Then, comparing D.4 and D.6 yields

$$c_1 \leq \frac{\beta d(\hat{p}_A, p_2^*(\ell_1)) + (1+\beta)I_1}{1 - \beta + \beta\delta}, \tag{D.10}$$

where I_1 satisfies D.8.

Now suppose that D.5 does not hold so that dictator A would not be reforming in $t = 2$ regardless of whether he reforms in $t = 1$ or not. Since he never pays c_2 in this scenario, there exist no future savings from disinvestments and the optimal action is to choose $I_1 = 0$. Then, comparing D.4 and D.7 yields

$$c_1 \leq (\beta + \beta^2) d(\hat{p}_A, p_2^*(\ell_1)). \tag{D.11}$$

The above analysis indicates that there are three possible outcomes to the dictatorship problem, depending on the given parameters and the initial states of the world: Immediate reform in $t = 1$, delaying reform until $t = 2$, and foregoing reform altogether. Dictator A prefers immediate reform over delaying whenever D.10 holds, which is more likely if c_1 is low, $d(\hat{p}_A, p_2^*(\ell_1))$ is high, and δ is low. On the other hand, he prefers immediate reform over never reforming whenever D.11 holds. This condition has a similar relationship with c_1 and $d(\hat{p}_A, p_2^*(\ell_1)$ as D.10. However, note that δ no longer plays a role in this case.

Suppose we have

$$\frac{\beta d(\hat{p}_A, p_2^*(\ell_1)) + (1+\beta)I_1}{1-\beta+\beta\delta} \leq (\beta+\beta^2)d(\hat{p}_A, p_2^*(\ell_1)), \tag{D.12}$$

where I_1 satisfies D.8. If we have $d(\hat{p}_A, p_1^*(\ell_1)) > (1-\delta)c_1$, D.12 reduces to the following condition:

$$1 - \delta < \frac{1}{\beta}, \tag{D.13}$$

which has to hold for all values of δ and β.[2] Thus, we can summarize the results of the analysis as follows:

First, immediately reforming is the unique optimal outcome for dictator A whenever D.10 holds, which would imply that D.11 also holds. Second, delaying reform is

[2]If $d(\hat{p}_A, p_1^*(\ell_1)) < (1-\delta)c_1$, we can still get the same result by imposing some parameter restrictions.

optimal whenever

$$c_1 \in \left[\frac{\beta d(\hat{p}_A, p_2^*(\ell_1)) + (1+\beta) I_1}{1-\beta+\beta\delta}, (\beta+\beta^2) d(\hat{p}_A, p_2^*(\ell_1)) \right], \tag{D.14}$$

where I_1 again satisfies D.8. Finally, if c_1 is such that

$$c_1 > (\beta+\beta^2) d(\hat{p}_A, p_2^*(\ell_1)), \tag{D.15}$$

dictator A never reforms the executive constraints. This completes the proof of Proposition 1. □

Proof of Proposition 2. Suppose without loss of generality that $\hat{p}_i > 0$ and $\hat{p}_j < 0$ for $j \neq i$. Then, it follows that $p_2^i(\ell_2) > 0$ and $p_2^j(\ell_2) < 0$. Focusing on the first period incumbent i's institutional choice, the Lagrangian for 2.13 can be written as

$$L_i = -\beta q_A \mid \hat{p}_i - p_2^A(\ell_2) \mid -\beta q_B \mid \hat{p}_i - p_2^B(\ell_2) \mid + \lambda \ell_2. \tag{D.16}$$

The corresponding first-order conditions for this problem are $\ell_2 \geq 0$ and

$$-\beta q_A(\hat{p}_i - p_2^A(\ell_2)) \frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2} - \beta q_B(\hat{p}_i - p_2^B(\ell_2)) \frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2} + \lambda \leq 0; \tag{D.17}$$

$$\left[-\beta q_A(\hat{p}_i - p_2^A(\ell_2)) \frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2} - \beta q_B(\hat{p}_i - p_2^B(\ell_2)) \frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2} + \lambda \right] \ell_2 = 0. \tag{D.18}$$

Whenever $q_i > 0$, $\ell_2 = 0$ cannot be a solution. Therefore, I look for a solution where $\lambda > 0$. Then, D.17 holds with equality so that the following equation implicitly defines the optimal choice of ℓ_2:

$$q_A(\hat{p}_i - p_2^A(\ell_2))\frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2} + q_B(\hat{p}_i - p_2^B(\ell_2))\frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2} = 0. \tag{D.19}$$

In order to see how the equilibrium value of $\rho^i(\ell_1) = \ell_2$ responds to changes in q_i, fix agent A as the first period incumbent without loss of generality and implicitly differentiate D.19 with respect to q_A to get $\frac{d\ell_2}{dq_A}$:

$$\begin{aligned}&\left[(\hat{p}_A - p_2^A(\ell_2)) - q_A\frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2}\frac{d\ell_2}{dq_A}\right]\frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2} + q_A(\hat{p}_A - p_2^A(\ell_2))\frac{d^2 p_2^A(\ell_2)}{d(\ell_2)^2}\frac{d\ell_2}{dq_A} \\ &+ \left[(\hat{p}_A - p_2^B(\ell_2)) - q_B\frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2}\frac{d\ell_2}{dq_A}\right]\frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2} + q_B(\hat{p}_A - p_2^B(\ell_2))\frac{d^2 p_2^B(\ell_2)}{d(\ell_2)^2}\frac{d\ell_2}{dq_A} = 0.\end{aligned} \tag{D.20}$$

Incumbent i's policy choice $p_t^i(\ell_t)$ that minimizes the distance to his ideal policy $\hat{p}_i$ implies that the second derivative of $p_t^i(\ell_t)$ with respect to ℓ_t is zero. Therefore, D.20 reduces to

$$\begin{aligned}&\left[(\hat{p}_A - p_2^A(\ell_2)) - q_A\frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2}\frac{d\ell_2}{dq_A}\right]\frac{d \mid p_2^A(\ell_2) \mid}{d\ell_2} \\ &+ \left[(\hat{p}_A - p_2^B(\ell_2)) - q_B\frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2}\frac{d\ell_2}{dq_A}\right]\frac{d \mid p_2^B(\ell_2) \mid}{d\ell_2} = 0.\end{aligned} \tag{D.21}$$

Solving D.21 for $\frac{d\ell_2}{dq_A}$ yields

$$\frac{d\ell_2}{dq_A} = \frac{(\hat{p}_A - p_2^A(\ell_2))\frac{d|p_2^A(\ell_2)|}{d\ell_2} + (\hat{p}_A - p_2^B(\ell_2))\frac{d|p_2^B(\ell_2)|}{d\ell_2}}{q_B\left(\frac{dp_2^B(\ell_2)}{d\ell_2}\right)^2 + q_A\left(\frac{dp_2^A(\ell_2)}{d\ell_2}\right)^2}. \tag{D.22}$$

Since $\frac{d|p_2^k(\ell_2)|}{d\ell_2}$ is positive for $k = A, B$, we conclude that $\frac{d\ell_2}{dq_A}$ is positive. This completes the proof of Proposition 2. □

Proof of Propositions 3 and 4. Since there exists no institutional or investment decision in $t = 3$, I start characterizing the equilibrium from $t = 2$. First, consider incumbent i's investment decision in this period. Since $T = 3$, either incumbent type chooses $I_2 = 0$ so that $\gamma_2^i(\ell_2, c_2) = (1 - \delta)c_2$ for both $i = A, B$.

Second, consider incumbent i's institutional strategy. Given ℓ_2, his payoff from letting $\ell_3 = \ell_2$ is given by[3]

$$-\beta q_i d(\hat{p}_i, p_3^i(\ell_2)) - \beta q_j d(\hat{p}_i, p_3^j(\ell_2)), \tag{D.23}$$

where $j \neq i$. On the other hand, if incumbent i chooses $\theta_2^i(\ell_2, c_2) \neq \ell_2$, the optimal choice of ℓ_3 is derived using the analysis in Proposition 2 that leads to the implicit equation D.19. Specifically, the optimal ℓ_3 is given by the solution to

$$q_A(\hat{p}_i - p_3^A(\ell_3))\frac{d \mid p_3^A(\ell_3) \mid}{d\ell_3} + q_B(\hat{p}_i - p_3^B(\ell_3))\frac{d \mid p_3^B(\ell_3) \mid}{d\ell_3} = 0. \tag{D.24}$$

Then, his payoff from choosing $\ell_3 \neq \ell_2$ such that D.24 is satisfied is given by

$$-c_2 - \beta q_i d(\hat{p}_i, p_3^i(\ell_3)) - \beta q_j d(\hat{p}_i, p_3^j(\ell_3)). \tag{D.25}$$

Therefore, incumbent i reforms the executive constraints whenever D.25 evaluated at

[3] I suppress the period-1 policy payoff in order to reduce clutter since it only depends on the exogenously given ℓ_1.

the optimal ℓ_3 given in D.24 is at least as great as D.23, which implies

$$\frac{c_2}{\beta} \leq q_i[d(\hat{p}_i, p_3^i(\ell_2)) - d(\hat{p}_i, p_3^i(\ell_3))] + q_j[d(\hat{p}_i, p_3^j(\ell_2)) - d(\hat{p}_i, p_3^j(\ell_3))]. \tag{D.26}$$

Since $d(\hat{p}_i, p_3^j(\ell_2)) < d(\hat{p}_i, p_3^j(\ell_3))$ and $d(\hat{p}_i, p_3^i(\ell_2)) > d(\hat{p}_i, p_3^i(\ell_3))$ (because $\ell_3 \geq \ell_2$), D.26 reduces to

$$\frac{c_2}{\beta} \leq q_i d(p_3^i(\ell_2), p_3^i(\ell_3)) - q_j d(p_3^j(\ell_2), p_3^j(\ell_3)), \tag{D.27}$$

which can be written in a more concise form as

$$\frac{c_2}{\beta} \leq \epsilon_3^i(\ell_3). \tag{D.28}$$

Condition D.28 indicates that incumbent i reforms the executive constraints in $t = 2$ if the given cost c_2 is lower than the present value of the expected net policy benefit of reform in $t = 3$. This proves part 1 of Proposition 3.

Now consider $t = 1$ and let agent A denote the first-period incumbent without loss of generality. If incumbent A reforms the executive constraints in $t = 1$ so that $\theta_1^A(\ell_1, c_1) \neq \ell_1$, he will not reform again in $t = 2$ if re-elected, resulting in the following payoff:

$$-c_1 - I_1 - \beta q_A d(\hat{p}_A, p_2^A(\theta_1^A(\ell_1, c_1))) - \beta q_B d(\hat{p}_A, p_2^B(\theta_1^A(\ell_1, c_1))) \tag{D.29}$$

$$-\beta^2 q_A q_A d(\hat{p}_A, p_3^A(\theta_1^A(\ell_1, c_1))) - \beta^2 q_A q_B d(\hat{p}_A, p_3^B(\theta_1^A(\ell_1, c_1)))$$

$$-\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\theta_2^B(\theta_1^A(\ell_1, c_1), c_2))) - \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\theta_2^B(\theta_1^A(\ell_1, c_1), c_2))).$$

On the other hand, if he sets $\ell_2 = \ell_1$, then his dynamic payoff depends on whether he will choose $\ell_3 = \ell_2$ in $t = 2$ if he is re-elected. Suppose incumbent A chooses I_1 such that D.28 holds, i.e. that he would reform in $t = 2$ if he were re-elected. Then, his payoff from choosing $\ell_2 = \ell_1$ is given by

$$-I_1 - \beta q_A d(\hat{p}_A, p_2^A(\ell_1)) - \beta q_B d(\hat{p}_A, p_2^B(\ell_1)) - \beta q_A c_2 \tag{D.30}$$

$$-\beta^2 q_A q_A d(\hat{p}_A, p_3^A(\theta_2^A(\ell_1, c_2))) - \beta^2 q_A q_B d(\hat{p}_A, p_3^B(\theta_2^A(\ell_1, c_2)))$$

$$-\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\theta_2^B(\ell_1, c_2))) - \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\theta_2^B(\ell_1, c_2))).$$

In contrast, if D.28 does not hold, his payoff from choosing $\ell_2 = \ell_1$ becomes

$$-I_1 - \beta q_A d(\hat{p}_A, p_2^A(\ell_1)) - \beta q_B d(\hat{p}_A, p_2^B(\ell_1)) \tag{D.31}$$

$$-\beta^2 q_A q_A d(\hat{p}_A, p_3^A(\ell_1)) - \beta^2 q_A q_B d(\hat{p}_A, p_3^B(\ell_1))$$

$$-\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\theta_2^B(\ell_1, c_2))) - \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\theta_2^B(\ell_1, c_2))).$$

Since these payoffs depend on I_1, we first need to characterize its optimal choice in each scenario.

If the optimal I_1 satisfies D.28, comparing D.29 and D.30 yields the condition that needs to hold in order for incumbent A to reform the level of executive constraints in $t = 1$ as opposed to delay reform until $t = 2$ (and risk it since it is only with probability q_A that he will be back in office). In this range, I_1 needs to satisfy

$$I_1 \leq \beta q_A d(p_3^A(\ell_1), p_3^A(\theta_2^A(\ell_1, c_2))) - \beta q_B d(p_3^B(\ell_1), p_3^B(\theta_2^A(\ell_1, c_2))) - (1-\delta)c_1 \quad \text{(D.32)}$$

in order for the resulting c_2 to make it profitable for the potential period-2 incumbent A to reform in that period.[4] Based on D.28, inequality D.32 can be written more concisely as

$$I_1 \leq \epsilon_3^A(\theta_2^A(\ell_1, c_2)) - (1-\delta)c_1. \quad \text{(D.33)}$$

First, consider the case in which incumbent A would delay reform until $t = 2$ by letting $\ell_2 = \ell_1$. Then, maximizing the relevant payoff D.30 by choosing I_1 subject to D.33 yields the following first-order condition:

$$-1 - \beta q_A - \beta^2 q_A q_A(\hat{p}_A - p_3^A(\theta_2^A(\ell_1, c_2)))\frac{d \mid p_3^A(\theta_2^A(\ell_1, c_2)) \mid}{d\ell_3}\frac{d\theta_2^A(\ell_1, c_2)}{dc_2} \quad \text{(D.34)}$$

$$-\beta^2 q_A q_B(\hat{p}_A - p_3^B(\theta_2^A(\ell_1, c_2)))\frac{d \mid p_3^B(\theta_2^A(\ell_1, c_2)) \mid}{d\ell_3}\frac{d\theta_2^A(\ell_1, c_2)}{dc_2}$$

$$-\beta^2 q_B q_A(\hat{p}_A - p_3^A(\theta_2^B(\ell_1, c_2)))\frac{d \mid p_3^A(\theta_2^B(\ell_1, c_2)) \mid}{d\ell_3}\frac{d\theta_2^B(\ell_1, c_2)}{dc_2}$$

$$-\beta^2 q_B q_B(\hat{p}_A - p_3^B(\theta_2^B(\ell_1, c_2)))\frac{d \mid p_3^B(\theta_2^B(\ell_1, c_2)) \mid}{d\ell_3}\frac{d\theta_2^B(\ell_1, c_2)}{dc_2} \leq 0.$$

Since incumbent A would reform in $t = 2$ if re-elected in the range of I_1 under consideration, the term $\frac{d\theta_2^A(\ell_1, c_2)}{dc_2}$ is equal to zero. However, incumbent A can manipulate his

[4]Note that D.28 can only hold if incumbent A has not already reformed the executive constraints in $t = 1$. If not, since the optimal level of executive constraints is the same once the decision to reform has been made, we would have $\ell_1 \neq \ell_2 = \ell_3$, resulting in $c_2 < 0$. Since this cannot be true, it follows that an incumbent can find it profitable to reform in $t = 2$ only if he himself has not reformed in $t = 1$.

opponent's decision on whether to reform in $t = 2$ or not if in office by appropriately choosing I_1. Specifically, given that D.28 holds and focusing on the case in which $\ell_2 = \ell_1$, incumbent A's payoff from preventing his opponent from choosing $\ell_3 \neq \ell_2$ if agent B is the period-2 incumbent is given by

$$-I_1 - \beta q_A d(\hat{p}_A, p_2^A(\ell_1)) - \beta q_B d(\hat{p}_A, p_2^B(\ell_1)) - \beta q_A c_2 \tag{D.35}$$

$$-\beta^2 q_A q_A d(\hat{p}_A, p_3^A(\theta_2^A(\ell_1, c_2))) - \beta^2 q_A q_B d(\hat{p}_A, p_3^B(\theta_2^A(\ell_1, c_2)))$$

$$-\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\ell_1)) - \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\ell_1)),$$

whereas his payoff from letting agent B reform the executive constraints under the same scenario is given by D.30. In order to prevent agent B from choosing $\theta_2^B(\ell_1, c_2) \neq \ell_1$ if agent B were to become the period-2 incumbent, incumbent A needs to invest such that

$$\frac{c_2}{\beta} \geq \epsilon_3^B(\theta_2^B(\ell_1, c_2)). \tag{D.36}$$

Specifically, the amount of investment to prevent the potential period-2 incumbent B from choosing $\theta_2^B(\ell_1, c_2) \neq \ell_1$ must satisfy

$$I_1 \geq \beta q_B d(p_3^B(\ell_1), p_3^B(\theta_2^B(\ell_1, c_2))) - \beta q_A d(p_3^A(\ell_1), p_3^A(\theta_2^B(\ell_1, c_2))) - (1 - \delta) c_1. \tag{D.37}$$

Since incumbent A cannot manipulate the *level* of ℓ_3 but only whether reform is carried out or not, the minimum investment that achieves his desired goal is the optimal choice of I_1. Hence, if incumbent A wants to prevent the potential period-2 incumbent B from reforming $\ell_2 = \ell_1$, he chooses I_1 such that D.37 holds with equality. This proves part 1 of Proposition 4.

On the other hand, if incumbent A has no such goal, the only inequality I_1 needs to satisfy is D.33. Since the only effect of investment in this case is on incumbent A's future ability to reform the executive constraints, he chooses $I_1 = 0$.[5] Then, it is optimal for incumbent A to prevent agent B from potentially reforming the executive constraints whenever D.35, evaluated at the investment choice such that D.37 holds with equality, is greater than or equal to D.30 evaluated at $I_1 = 0$.

Still considering the case in which the optimal I_1 satisfies D.28, consider the institutional choice $\ell_2 \neq \ell_1$ by incumbent A. Since he never reforms the executive constraints twice, this scenario can never be optimal. Therefore, we conclude that if the I_1 choice is such that it is optimal for incumbent A to reform in $t = 2$ if he were re-elected, he will not reform in $t = 1$. This leaves D.30 and D.35 as the relevant payoffs whenever D.28 holds, depending on whether agent B will be blocked. This comparison yields the following simplified condition:

$$(1 + \beta q_A)[(1 - \delta)c_1 - \epsilon_3^B(\theta_2^B(\ell_1, c_2))] \tag{D.38}$$

$$\beta^2 q_B q_B[d(\hat{p}_A, p_3^B(\theta_2^B(\ell_1, c_2))) - d(\hat{p}_A, p_3^B(\ell_1))]$$

$$\geq$$

$$\beta^2 q_B q_A[d(\hat{p}_A, p_3^A(\ell_1)) - d(\hat{p}_A, p_3^A(\theta_2^B(\ell_1, c_2)))],$$

[5]Note that there would not be disinvestments in this case as he can recoup the disinvestment only with probability q_A.

where $c_2 = \beta\epsilon_3^B(\theta_2^B(\ell_1, c_2))$ based on D.36. Given $\theta_1^A(\ell_1, c_1) = \ell_1$ and that he would reform in $t = 2$ if re-elected, the condition D.38 indicates that incumbent A is more likely to prevent agent B from potentially reforming the executive constraints in $t = 2$ if the cost of preventing opponent B is low (first line in D.38), the policy cost in $t = 3$ in case of agent B's election of allowing the potential period-2 incumbent B to set ℓ_3 is high (second line in D.38), and the policy benefit to incumbent A from enjoying the weaker ℓ_3 set by his opponent B if incumbent A retakes office from agent B in $t = 3$ is low (third line in D.38).

Now, consider the case in which I_1 is such that D.28 does not hold. In this scenario, comparing D.29 and D.31 yields the condition that needs to hold in order for incumbent A to reform the level of executive constraints in $t = 1$ as opposed to forgoing reform altogether. As before, we first characterize the optimal levels of I_1.

Focusing on the range of I_1 for which D.28 does not hold, the optimal I_1 needs to satisfy

$$I_1 \geq \beta q_A d(p_3^A(\ell_1), p_3^A(\theta_2^A(\ell_1, c_2))) - \beta q_B d(p_3^B(\ell_1), p_3^B(\theta_2^A(\ell_1, c_2))) - (1-\delta)c_1, \quad \text{(D.39)}$$

which can again be re-written more concisely as

$$I_1 \geq \epsilon_3^A(\theta_2^A(\ell_1, c_2)) - (1-\delta)c_1. \quad \text{(D.40)}$$

However, since $\theta_2^A(\ell_1, c_2) = \ell_1$ in this scenario, D.40 reduces to

$$I_1 \geq -(1-\delta)c_1. \tag{D.41}$$

First, consider the case in which incumbent A does not reform in $t = 1$. Maximizing the relevant payoff D.31 by choosing I_1 subject to D.41 yields the following first-order condition, which already reflects the fact that incumbent A's period-2 institutional decision is determined under the considered case that D.28 does not hold:

$$-1 - \beta^2 q_B q_A(\hat{p}_A - p_3^A(\theta_2^B(\ell_1, c_2)))\frac{d \mid p_3^A(\theta_2^B(\ell_1, c_2)) \mid}{d\ell_3}\frac{d\theta_2^B(\ell_1, c_2)}{dc_2} \tag{D.42}$$

$$-\beta^2 q_B q_B(\hat{p}_A - p_3^B(\theta_2^B(\ell_1, c_2)))\frac{d \mid p_3^B(\theta_2^B(\ell_1, c_2)) \mid}{d\ell_3}\frac{d\theta_2^B(\ell_1, c_2)}{dc_2} \leq 0.$$

Given that D.28 does not hold and focusing on the case in which $\ell_2 = \ell_1$, incumbent A's payoff from preventing agent B from choosing $\ell_3 \neq \ell_2$ if agent B is the period-2 incumbent is given by

$$-I_1 - \beta q_A d(\hat{p}_A, p_2^A(\ell_1)) - \beta q_B d(\hat{p}_A, p_2^B(\ell_1)) \tag{D.43}$$

$$-\beta^2 q_A q_A d(\hat{p}_A, p_3^A(\ell_1)) - \beta^2 q_A q_B d(\hat{p}_A, p_3^B(\ell_1))$$

$$-\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\ell_1)) - \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\ell_1)),$$

whereas his payoff from letting agent B pick his desired executive constraints is given by D.31. By a similar logic, in order to prevent the potential period-2 incumbent

B from choosing $\theta_2^B(\ell_1, c_2) \neq \ell_1$, incumbent A needs to invest an amount such that D.36 is satisfied. Specifically, incumbent A must choose I_1 such that D.37 holds with equality. On the other hand, if incumbent A chooses to let agent B reform the executive constraints according to his preferences, he would neither invest nor disinvest, since he doesn't reform in $t = 2$. Then, incumbent A will prevent the potential period-2 incumbent B from reforming the executive constraints whenever D.43, evaluated at the investment choice where D.37 holds with equality, is greater than or equal to D.31, evaluated at $I_1 = 0$. This comparison yields

$$\epsilon_3^B(\theta_2^B(\ell_1, c_2)) - (1-\delta)c_1 \tag{D.44}$$

$$\leq$$

$$\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\theta_2^B(\ell_1, c_2))) + \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\theta_2^B(\ell_1, c_2))),$$

where $c_2 = \beta\epsilon_3^B(\theta_2^B(\ell_1, c_2))$. Condition D.44 indicates that incumbent A is more likely to prevent agent B's potential reform if the cost of doing so and the policy benefit from letting agent B choose $\ell_3 = \theta_2^B(\ell_1, c_2)$ are low.[6]

Second, still keeping the assumption that D.28 does not hold given the chosen I_1, consider the case in which $\ell_2 \neq \ell_1$. Maximizing the relevant payoff D.29 by choosing I_1 subject to D.41 yields the following first-order condition, again reflecting the fact that investment does not affect incumbent A's institutional decision in the case under

[6]Specifically, the negative of the expected distance of period-3 policy from $\hat{p}_A$ is high when the period-3 executive constraints are set according to agent B's preferences.

consideration:

$$-1-\beta^2 q_B q_A(\hat{p}_A - p_3^A(\theta_2^B(\theta_1^A(\ell_1,c_1),c_2)))\frac{d\mid p_3^A(\theta_2^B(\theta_1^A(\ell_1,c_1),c_2))\mid}{d\ell_3}\frac{d\theta_2^B(\theta_1^A(\ell_1,c_1),c_2)}{dc_2} \tag{D.45}$$

$$-\beta^2 q_B q_B(\hat{p}_A - p_3^B(\theta_2^B(\theta_1^A(\ell_1,c_1),c_2)))\frac{d\mid p_3^B(\theta_2^B(\theta_1^A(\ell_1,c_1),c_2))\mid}{d\ell_3}\frac{d\theta_2^B(\theta_1^A(\ell_1,c_1),c_2)}{dc_2} \leq 0.$$

Given that D.28 does not hold and focusing on the case in which there is immediate reform, incumbent A's payoff from preventing the potential period-2 incumbent B from choosing $\theta_2^B(\theta_1^A(\ell_1,c_1),c_2) \neq \theta_1^A(\ell_1,c_1)$ is given by

$$-c_1 - I_1 - \beta q_A d(\hat{p}_A, p_2^A(\theta_1^A(\ell_1,c_1))) - \beta q_B d(\hat{p}_A, p_2^B(\theta_1^A(\ell_1,c_1))) \tag{D.46}$$

$$-\beta^2 q_A q_A d(\hat{p}_A, p_3^A(\theta_1^A(\ell_1,c_1))) - \beta^2 q_A q_B d(\hat{p}_A, p_3^B(\theta_1^A(\ell_1,c_1)))$$

$$-\beta^2 q_B q_A d(\hat{p}_A, p_3^A(\theta_1^A(\ell_1,c_1))) - \beta^2 q_B q_B d(\hat{p}_A, p_3^B(\theta_1^A(\ell_1,c_1))).$$

On the other hand, his payoff from letting agent B choose $\theta_2^B(\theta_1^A(\ell_1,c_1),c_2)$ is given by D.29. The optimal investment set by incumbent A that would prevent the potential period-2 incumbent B from reforming the executive constraints and the amount that would let him choose his desired ℓ_3 are the same as in the above case that considered $\ell_2 = \ell_1$. Hence, comparing D.46 with D.29 yields the following condition that needs to hold in order for incumbent A to prevent the potential incumbent B from designating ℓ_3 as he wishes, given D.28 does not hold and $\ell_2 \neq \ell_1$:

$$(1-\delta)c_1 - \epsilon_3^B(\theta_2^B(\theta_1^A(\ell_1,c_1),c_2)) \tag{D.47}$$

$$\beta^2 q_B q_B[d(\hat{p}_A, p_3^B(\theta_2^B(\theta_1^A(\ell_1, c_1), c_2))) - d(\hat{p}_A, p_3^B(\theta_1^A(\ell_1, c_1)))]$$

$$\geq$$

$$\beta^2 q_B q_A[d(\hat{p}_A, p_3^A(\theta_1^A(\ell_1, c_1))) - d(\hat{p}_A, p_3^A(\theta_2^B(\theta_1^A(\ell_1, c_1), c_2)))].$$

The above inequality has a similar interpretation as the previous conditions D.38 and D.44. Since all three conditions D.38, D.44, and D.47 yield the same conclusion, I focus on D.38 for its relative simplicity in order to derive the conditions on the parameters of the model that need to hold in order for incumbent A to find it profitable to block his opponent B's potential institutional decision in $t = 2$ by investing the necessary amount I_1 that makes D.37 hold with equality.

Substitute the expression for $\epsilon_3^B(\theta_2^B(\ell_1, c_2))$ into D.38 and re-arrange to get

$$(1+\beta q_A)[(1-\delta)c_1 - \beta q_A d(p_3^A(\ell_1), p_3^A(\theta_2^B(\ell_1, c_2))) + q_B d(p_3^B(\ell_1), p_3^B(\theta_2^B(\ell_1, c_2)))] \quad \text{(D.48)}$$

$$+\beta^2 q_B[q_B d(p_3^B(\theta_2^B(\ell_1, c_2))), p_3^B(\ell_1)) + q_A d(p_3^A(\theta_2^B(\ell_1, c_2))), p_3^A(\ell_1))] \geq 0,$$

where $c_2 = \beta\epsilon_3^B(\theta_2^B(\ell_1, c_2))$ based on D.36. Further simplifying yields

$$(1+\beta q_A)(1-\delta)c_1 - \beta q_A d(p_3^A(\theta_2^B(\ell_1, c_2))), p_3^A(\ell_1))[1 + \beta(q_A - q_B)] \quad \text{(D.49)}$$

$$+\beta q_B d(p_3^B(\theta_2^B(\ell_1, c_2))), p_3^B(\ell_1))[1+\beta] \geq 0.$$

To re-iterate, this is the condition that needs to hold in order for incumbent A to prevent agent B from reforming the executive constraints in a particular scenario. There are two other conditions pertaining to different scenarios, but they yield the same relationships as D.49. This condition implies that incumbent A becomes more likely to block agent B's future institutional decision as q_B and $d(p_3^B(\theta_2^B(\ell_1, c_2))), p_3^B(\ell_1))$ increase and $d(p_3^A(\theta_2^B(\ell_1, c_2))), p_3^A(\ell_1))$ decreases. This proves part 2 of Proposition 4.

In order to complete the proof of Proposition 3, we now return to analyzing the conditions that need to hold in order for incumbent A to prefer one reform path over another. Note that there are three possible decisions he can make in $t = 1$: Reform the executive constraints immediately, delay reform until $t = 2$ with the expectation of re-election, or never reform. Since an incumbent never reforms twice, we need to focus on the cases in which D.28 does not hold in order to see when immediate reform is optimal for incumbent A.

Suppose D.47 holds such that incumbent A finds it profitable to prevent his opponent's institutional decision in $t = 2$. Then, to see when immediate reform is preferred by incumbent A over foregoing reform altogether, we first need to compare D.46 with D.43, which is the payoff from letting $\ell_2 = \ell_1$ and preventing the opponent B. This comparison yields

$$c_1 + \beta q_B d(p_2^B(\ell_1), p_2^B(\theta_1^A(\ell_1, c_1)))[1 + \beta] \tag{D.50}$$

$$\leq$$

$$\beta q_A d(p_2^A(\theta_1^A(\ell_1, c_1)), p_2^A(\ell_1))[1 + \beta].$$

Second, we need to compare D.46 with D.31, which is the payoff from letting $\ell_2 = \ell_1$

and not preventing agent B's institutional decision. This comparison yields

$$c_1 + \beta q_B d(p_2^B(\ell_1), p_2^B(\theta_1^A(\ell_1, c_1)))[1 + \beta q_A] \tag{D.51}$$

$$\leq$$

$$\beta q_A d(p_2^A(\theta_1^A(\ell_1, c_1)), p_2^A(\ell_1))[1 + \beta q_A].$$

Conditions D.50 and D.51 both imply that incumbent A prefers immediate reform over foregoing reform altogether whenever c_1, q_B, and the policy cost to him of reforming in case agent B becomes the incumbent are low, and q_A and the policy benefit of reforming in case of his re-election is high. On the other hand, if we suppose D.47 does not hold so that incumbent A does not invest sufficiently to block his opponent B, we need to compare D.29 with the same payoffs D.43 and D.31, which yield the same relationships and hence are not repeated here.

Now consider incumbent A's decision between delaying reform until $t = 2$ and foregoing reform altogether. Suppose D.38 holds so that incumbent A would prevent his opponent from choosing $\ell_3 \neq \ell_2$. Then, we compare D.35 with D.43 and D.31 as in the above paragraph in order to obtain the condition that needs to hold for incumbent A to prefer delaying reform until $t = 2$ as opposed to never reforming. These comparisons respectively yield the following conditions:

$$(1 - \delta)c_1 + I_1 + \beta q_B d(p_3^B(\theta_2^A(\ell_1, c_2)), p_3^B(\ell_1)) \tag{D.52}$$

$$\leq$$

$$\beta q_A d(p_3^A(\ell_1), p_3^A(\theta_2^A(\ell_1, c_2)))$$

and

$$I_1 + \beta q_A[(1-\delta)c_1 + I_1] \tag{D.53}$$

$$+\beta^2 q_A q_B[d(p_3^B(\ell_1), p_3^B(\theta_2^A(\ell_1, c_2))) + d(p_3^A(\ell_1), p_3^A(\theta_2^B(\ell_1, c_2)))]$$

$$\leq$$

$$\beta^2 q_A q_A d(p_3^A(\ell_1), p_3^A(\theta_2^A(\ell_1, c_2))) + \beta^2 q_B q_B d(p_3^B(\ell_1), p_3^B(\theta_2^B(\ell_1, c_2))),$$

where I_1 is again such that D.37 holds with equality. Conditions D.52 and D.53 both imply that incumbent A prefers to delay reform over foregoing reform altogether whenever c_1 and q_B are low. On the other hand, if we suppose that D.38 does not hold so that incumbent A will not block his opponent, then we compare the same two payoffs D.43 and D.31 with D.30. As these comparisons yield a similar relationship as observed in D.52 and D.53, this exercise is not repeated here. These comparisons together prove part 2 of Proposition 3.

Finally, to see when immediate reform may be preferred to delaying until $t = 2$, we compare D.50 versus D.52, and D.51 versus D.53. These comparisons indicate that immediate reform is more likely to be preferred over delaying whenever $(1-\delta)c_1 + I_1$ is high. Together with the above inequalities, this proves part 3 of Proposition 3 and hence completes the proofs of Propositions 3 and 4. □

Appendix E

Proofs for Chapter 3

Proof of Lemma 1. Based on 3.8 and 3.9, the first-order conditions for the parties' optimal campaign spending choices are given by

$$\epsilon_1(z)\left[\frac{\zeta_2(z)}{(\zeta_1(z)+\zeta_2(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_1(z)>w_1\\ =0 & \text{if } \zeta_1(z)\in[0,w_1]\\ \leq 0 & \text{if } \zeta_1(z)=0;\end{cases}\tag{E.1}$$

and

$$\epsilon_2(z)\left[\frac{\zeta_1(z)}{(\zeta_1(z)+\zeta_2(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_2(z)>w_2\\ =0 & \text{if } \zeta_2(z)\in[0,w_2]\\ \leq 0 & \text{if } \zeta_2(z)=0.\end{cases}\tag{E.2}$$

Solving for $\zeta_1(z)$ and $\zeta_2(z)$ based on E.1 and E.2 implies that the unique pair of campaign spending rules $(\zeta_1(z),\zeta_2(z))$ is given by one of the following four equilibrium candidates, depending on the outcome of the bargaining stage:

1. $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$ if and only if $\epsilon_1(z) \geq \frac{(w_1+w_2)^2}{w_2}$ and $\epsilon_2(z) \geq \frac{(w_1+w_2)^2}{w_1}$.

2. $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$ if and only if $\epsilon_1(z) \geq \frac{w_1\epsilon_2(z)}{\sqrt{w_1\epsilon_2(z)}-w_1}$ and $\epsilon_2(z) \leq \frac{(w_1+w_2)^2}{w_1}$.

3. $(\zeta_1(z), \zeta_2(z)) = (\sqrt{w_2\epsilon_1(z)} - w_2, w_2)$ if and only if $\epsilon_1(z) \leq \frac{(w_1+w_2)^2}{w_2}$ and $\epsilon_2(z) \geq \frac{w_2\epsilon_1(z)}{\sqrt{w_2\epsilon_1(z)}-w_2}$.

4. $(\zeta_1(z), \zeta_2(z)) = (\frac{\epsilon_1(z)^2\epsilon_2(z)}{[\epsilon_1(z)+\epsilon_2(z)]^2}, \frac{\epsilon_1(z)\epsilon_2(z)^2}{[\epsilon_1(z)+\epsilon_2(z)]^2})$ if and only if $\frac{\epsilon_1(z)^2\epsilon_2(z)}{[\epsilon_1(z)+\epsilon_2(z)]^2} < w_1$ and $\frac{\epsilon_1(z)\epsilon_2(z)^2}{[\epsilon_1(z)+\epsilon_2(z)]^2} < w_2$.

Notice that if the challenge stage equilibrium is such that $\zeta_k(z) = w_k$, then $\zeta_k(z)$ is constant in the value of $\epsilon_k(z)$. On the other hand, if $\zeta_k(z) = \sqrt{w_{-k}\epsilon_k(z)} - w_{-k}$ or if we have an interior equilibrium as characterized in item four above, then $\zeta_k(z)$ is increasing in the value of $\epsilon_k(z)$. This is straightforward to see for the first case. To see this for the interior challenge stage equilibrium, differentiate $\zeta_k(z)$ as characterized in item four with respect to the value of $\epsilon_k(z) \equiv \bar{\epsilon}_k$ to get

$$\frac{(2\bar{\epsilon}_k\bar{\epsilon}_{-k})(\bar{\epsilon}_k + \bar{\epsilon}_{-k})^2 - (2\bar{\epsilon}_k^2\bar{\epsilon}_{-k})(\bar{\epsilon}_k + \bar{\epsilon}_{-k})}{(\bar{\epsilon}_k + \bar{\epsilon}_{-k})^4}, \tag{E.3}$$

whose both numerator and denominator are positive. Hence, we conclude that the interior equilibrium level of campaign spending of each party k is increasing in the value of $\epsilon_k(z)$. This completes the proof of Lemma 1.

$\square$

Proof of Proposition 1. Using backward induction, I first characterize the equilibrium acceptance strategy $a_2(z)$ of party 2 for any given proposal z.

If party 2 accepts party 1's proposal z, its payoff would be given by $u_2(z)$ with certainty. Since it is risk-neutral, party 2 will accept any offer that yields a sure payoff of $u_2(z)$ that is at least as great as its expected payoff from the challenge stage equilibrium that would be observed based on $\epsilon_1(z)$ and $\epsilon_2(z)$.

Given w_1, w_2, and the status-quo bill s, suppose party 1 makes an offer z such that $\epsilon_k(z) \geq \frac{(w_1+w_2)^2}{w_{-k}}$ for both k. If rejected, this offer would imply $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$. Therefore, given $\rho_1(z) = Z$ and $\rho_2(z) = S$ in any challenge stage equilibrium, this proposal z implies an expected payoff for party 2 from the challenge stage given by

$$u_2(s) + \left(\frac{w_1}{w_1 + w_2}\right)[u_2(z) - u_2(s)] - w_2. \tag{E.4}$$

Comparing the sure payoff $u_2(z)$ with E.4 implies that party 2 accepts z if and only if $u_2(z) \geq u_2(s) - (w_1 + w_2)$, which can also be written as $\epsilon_2(z) \leq w_1 + w_2$. However, since the proposal z under consideration is such that $\epsilon_2(z) \geq \frac{(w_1+w_2)^2}{w_1}$ and $\frac{(w_1+w_2)^2}{w_1} > w_1 + w_2$, the acceptance criteria can never be satisfied. Therefore, any proposal z that would pave the way for a challenge stage with $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$ if rejected will be rejected by party 2.

Second, suppose party 1 makes an offer z such that the conditions for a challenge stage equilibrium in which $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1 \epsilon_2(z)} - w_1)$ as listed in item two in the proof of Lemma 1 are satisfied. This offer implies the following expected payoff for party 2 from the challenge stage:

$$u_2(s) + \sqrt{\frac{w_1}{\epsilon_2(z)}}[u_2(z) - u_2(s)] - \sqrt{w_1\epsilon_2(z)} + w_1. \tag{E.5}$$

Comparing the sure payoff $u_2(z)$ with E.5 implies that party 2 accepts z if and only if

$$u_2(z) \geq u_2(s) + \frac{w_1\sqrt{\epsilon_2(z)} - \sqrt{w_1}\epsilon_2(z)}{\sqrt{\epsilon_2(z)} - \sqrt{w_1}}, \tag{E.6}$$

where the last term is negative since $\zeta_2(z) = \sqrt{w_1\epsilon_2(z)} - w_1$. Re-arranging E.6 yields

$$\epsilon_2(z) \leq \frac{(\sqrt{w_1\epsilon_2(z)} - w_1)\sqrt{\epsilon_2(z)}}{\sqrt{\epsilon_2(z)} - \sqrt{w_1}}, \tag{E.7}$$

which reduces to $\epsilon_2(z) \leq w_1$. Therefore, party 2 will accept any proposal z that would imply a subsequent challenge stage with $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$ as long as $\epsilon_2(z) \leq w_1$.

Third, suppose party 1 makes an offer z such that the equilibrium campaign spending if z were rejected is given by $(\zeta_1(z), \zeta_2(z)) = (\sqrt{w_2\epsilon_1(z)} - w_2, w_2)$. The implied expected challenge stage payoff for party 2 is given in this case by

$$u_2(s) + \frac{\sqrt{w_2\epsilon_1(z)} - w_2}{\sqrt{w_2\epsilon_1(z)}}[u_2(z) - u_2(s)] - w_2. \tag{E.8}$$

Then, party 2 accepts any offer z that yields a sure payoff of $u_2(z)$ that is at least as great as E.8, which reduces to the condition that z must satisfy $\epsilon_2(z) \leq \sqrt{w_2\epsilon_1(z)}$. However, since the proposal z under consideration is such that $\epsilon_2(z) \geq \frac{w_2\epsilon_1(z)}{\sqrt{w_2\epsilon_1(z) - w_2}}$, the acceptance criteria can never be satisfied, because $\sqrt{w_2\epsilon_1(z)} < \frac{w_2\epsilon_1(z)}{\sqrt{w_2\epsilon_1(z)-w_2}}$. Therefore, party 2 will reject all offers that would subsequently lead to a challenge stage with $(\zeta_1(z), \zeta_2(z)) = (\sqrt{w_2\epsilon_1(z)} - w_2, w_2)$.

Finally, suppose party 1's offer z is such that the equilibrium campaign spending in any challenge to z would be given by the interior equilibrium as listed in item four in the proof of Lemma 1. Constructing the expected payoff from the challenge stage as in the above cases yields the condition that z must satisfy $\epsilon_2(z) \leq \zeta_1(z) + \zeta_2(z)$ in order to be accepted by party 2. Plugging in the equilibrium values of $\zeta_1(z)$ and $\zeta_2(z)$ into this condition yields

$$\epsilon_2(z) \leq \frac{\epsilon_1(z)\epsilon_2(z)}{\epsilon_1(z) + \epsilon_2(z)}, \tag{E.9}$$

which reduces to the condition that party 2 will accept any offer z for which $u_2(z) \geq u_2(s)$ whenever the subsequent challenge stage equilibrium if z is rejected would be an interior one.

Bringing together the above characterization of party 2's acceptance rules for each possible challenge stage equilibrium, we observe that any proposal z for which $\zeta_2(z) = w_2$ is rejected (although these are not the only offers that will be rejected). In addition, whenever z is such that $\zeta_2(z) < w_2$, party 2 accepts any offer for which $\epsilon_2(z) \leq w_1$ if $\zeta_1(z) = w_1$ and any offer for which $u_2(z) \geq u_2(s)$ if $\zeta_1(z) < w_1$. This proves part 1 of Proposition 1.

Given the equilibrium acceptance strategy of party 2 for any proposal z, I now solve for party 1's optimal proposals. In the rest of Proposition 1, part 2 solves for the best way to induce unanimity, whereas part 3 solves for the optimal proposal that would push the game to the challenge stage.

Suppose that party 1 will make an offer that will get party 2's acceptance, thereby avoiding a challenge stage. The proof of part 1 indicated that there exist two methods with which party 1 can induce unanimity in the parliament: By offering z such that a) $\epsilon_1(z) \geq \frac{w_1\epsilon_2(z)}{\sqrt{w_1\epsilon_2(z)-w_1}}$ and $\epsilon_2(z) \leq w_1$; or b) $\frac{\epsilon_k(z)^2\epsilon_{-k}(z)}{(\epsilon_1(z)+\epsilon_2(z))^2} < w_k$ for both k and $u_2(z) \geq u_2(s)$. Since the first method implies that party 1 only needs to propose a z for which $u_2(z) = u_2(s) - w_1$, whereas the second method requires $u_2(z) = u_2(s)$ for acceptance, party 1 would choose the first method if it wanted to induce unanimity.[1]

To solve for the specifics of this offer, party 1 maximizes $u_1(z)$ subject to party 1's acceptance constraint $u_2(z) \geq u_2(s) - w_1$ and the technical constraint $z \in [0,1] \times Y$. The Lagrangian of this problem can be written as follows:

$$L = -(x-\hat{x}_1)^2 + \alpha y_1 + \lambda_1[-(x-\hat{x}_2)^2 + \alpha(1-y_1) - u_2(s) + w_1] \tag{E.10}$$

$$+\mu_1 x - \mu_2(x-1) + \gamma_1 y_1 - \gamma_2(y-1).$$

The first-order conditions for E.10 are $x \in [0,1]$, $y_1 \in [0,1]$, $\lambda_1 \geq 0$, $\mu_1 \geq 0$, $\mu_2 \geq 0$, $\gamma_1 \geq 0$, $\gamma_2 \geq 0$,

$$-2(x-\hat{x}_1) - 2\lambda_1(x-\hat{x}_2) + (\mu_1 - \mu_2) \leq 0; \tag{E.11}$$

$$[-2(x-\hat{x}_1) - 2\lambda_1(x-\hat{x}_2) + (\mu_1 - \mu_2)]x = 0; \tag{E.12}$$

[1] To rule out extreme parameter cases such that no proposal z would justify $\zeta_1(z) = w_1$, I impose the restriction that if $\epsilon_2(z) \leq \frac{(w_1+w_2)^2}{w_1}$, then $\epsilon_1(z) \geq \frac{(w_1)^2(w_1+w_2)^2}{w_2}$, which implies $\frac{\epsilon_1(z)}{\epsilon_2(z)} \geq (w_1)^2$. This assumption ensures that we can restrict our attention to the first of the two methods for inducing unanimity.

$$\alpha - \lambda_1 \alpha + (\gamma_1 - \gamma_2) \leq 0; \tag{E.13}$$

$$[\alpha - \lambda_1 \alpha + (\gamma_1 - \gamma_2)] y_1 = 0; \tag{E.14}$$

$$-(x - \hat{x}_2)^2 + \alpha(1 - y_1) - u_2(s) + w_1 \geq 0; \tag{E.15}$$

$$[-(x - \hat{x}_2)^2 + \alpha(1 - y_1) - u_2(s) + w_1] \lambda_1 = 0; \tag{E.16}$$

along with $\mu_1 x = 0$; $\mu_2(1 - x) = 0$; $\gamma_1 y_1 = 0$; and $\gamma_2(1 - y_1) = 0$. An interior solution to this problem entails $\mu_1 = \mu_2 = \gamma_1 = \gamma_2 = 0$, and $\lambda_1 = 1$ based on E.14, yielding

$$x = \frac{\hat{x}_1 + \hat{x}_2}{2}. \tag{E.17}$$

Solving for y_2 using the fact that party 1 will not make an offer z that gives party 2 any higher utility than is needed for acceptance, $u_2(z) = u_2(s) - w_1$ implies

$$y_2 = \alpha^{-1} \left[\left(\frac{\hat{x}_1 + \hat{x}_2}{2} \right)^2 - (q - \hat{x}_2)^2 + \alpha y_2^q - w_1 \right]. \tag{E.18}$$

Therefore, an equilibrium proposal z characterized by the ideology component in E.17 and the rent component with y_2 as given in E.18 and $y_1 = 1 - y_2$ induces an optimal unanimity outcome for party 1. Specifically, $y_1 = 1 - y_2$ is given by

$$y_1 = \alpha^{-1} \left[- \left(\frac{\hat{x}_1 + \hat{x}_2}{2} \right)^2 + (q - \hat{x}_2)^2 + \alpha y_1^q + w_1 \right]. \tag{E.19}$$

Therefore, the difference between the rent shares of the two parties is given by

$$y_1 - y_2 = \alpha^{-1}\left[-\frac{(\hat{x}_1 + \hat{x}_2)^2}{2} + 2(q - \hat{x}_2)^2 + \alpha(y_1^q - y_2^q) + 2w_1\right]. \tag{E.20}$$

Notice that this difference increases as party 2's status-quo payoff decreases and w_1 increases.

Now consider possible corner solutions to this maximization problem. First, I claim that there exists no solution with $x = 0$ or $x = 1$. To see this, first let $\mu_1 > 0$ and $\mu_2 = \gamma_1 = \gamma_2 = 0$. This yields $\lambda_1 = 1$ as before, resulting in the equality $2\hat{x}_1 + 2\hat{x}_2 + \mu_1 = 0$. Since this would imply $\mu_1 < 0$, the desired result is achieved. Second, let $\mu_2 > 0$ and $\mu_1 = \gamma_1 = \gamma_2 = 0$. This situation yields the equality $-4 + 2\hat{x}_1 + 2\hat{x}_2 - \mu_2 = 0$, implying that μ_2 must be negative. Hence, we can conclude that the optimal ideology component of z must be such that $x \in (0, 1)$.

Second, I claim that solutions with $y_1 = 0$ or $y_1 = 1$ are possible for certain values of α. Suppose $\gamma_1 > 0$ and $\gamma_2 = 0$. With $\mu_1 = \mu_2 = 0$, this yields $\alpha - \lambda_1\alpha + \gamma_1 < 0$, or $\lambda_1 > \frac{\alpha+\gamma_1}{\alpha}$. Then, the condition $(x - \hat{x}_1) + \lambda_1(x - \hat{x}_2) = 0$ implies

$$\frac{x - \hat{x}_1}{x - \hat{x}_2} = -\lambda_1 < -\left(1 + \frac{\gamma_1}{\alpha}\right), \tag{E.21}$$

which can hold for small values of α, yielding $y_1 = 0$. In this situation, party 1 chooses x closer to $\hat{x}_1$. Likewise, letting $\gamma_2 > 0$ implies

$$\frac{x - \hat{x}_1}{x - \hat{x}_2} = -\lambda_1 < -\left(1 - \frac{\gamma_2}{\alpha}\right), \tag{E.22}$$

which can hold for larger values of the parameter α, yielding $y_1 = 1$. Here, party 1 chooses x closer to $\hat{x}_2$ in order to secure party 2's acceptance. This concludes the proof of part 2 of Proposition 1.

For part 3, suppose that party 1 will make an offer that will lead to a challenge on the equilibrium path. Note that of the four methods with which party 1 can push the bill into a challenge as summarized in part 1, two of these methods involve proposals that would imply $\zeta_2(z) = w_2$ in the referendum. In this case, the optimal z is such that $x = \hat{x}_1$, $y_1 = 1$, and $y_2 = 0$. This is due to the fact that once party 2 starts spending a constant sum of w_2, the proposal z no longer affects the probability of winning for party 1. Therefore, party 1 maximizes its expected payoff from the referendum by maximizing the value of $\epsilon_1(z)$.

To see when inducing a challenge stage equilibrium with $\zeta_2(z) < w_2$ would be preferred to one with $\zeta_2(z) = w_2$, I focus on the challenge stage equilibrium in which $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$, which arises if the rejected proposal z is such that $\epsilon_1(z) \geq \frac{w_1\epsilon_2(z)}{\sqrt{w_1\epsilon_2(z)} - w_1}$ and $\epsilon_2(z) \in \left(w_1, \frac{(w_1+w_2)^2}{w_1}\right)$.[2] For any proposal z that satisfies these conditions, the expected payoff to party 1 from this challenge stage equilibrium is given by

$$u_1(s) + \sqrt{\frac{w_1}{\epsilon_2(z)}}\epsilon_1(z) - w_1, \tag{E.23}$$

maximizing which subject to the above conditions yields $x = \frac{\hat{x}_1+\hat{x}_2}{2}$.

Party 1 prefers this challenge stage equilibrium with proposal z to the one in which

[2]This is also justified by the parameter restriction imposed in Footnote 1.

$(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$ whenever

$$\sqrt{\frac{w_1}{\epsilon_2(z)}}\epsilon_1(z) \geq \frac{w_1}{w_1 + w_2}(\alpha - u_1(s)). \tag{E.24}$$

Note that since z is such that $\epsilon_2(z) \in \left(w_1, \frac{(w_1+w_2)^2}{w_1}\right)$, the probability of winning is always at least as high for party 1 on the left-hand side of E.24 as on the right-hand side of it. Therefore, this inequality needs the proposal z that would induce the challenge stage equilibrium with $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$ to be such that

$$\epsilon_1(z) \in \left[\frac{w_1}{w_1 + w_2}(\alpha - u_1(s)), (\alpha - u_1(s))\right] \tag{E.25}$$

in order to hold. Therefore, if the optimal proposal that would induce this challenge implies $\epsilon_1(z) < \frac{w_1}{w_1+w_2}(\alpha - u_1(s))$, party 1 prefers the challenge stage equilibrium with $\zeta_2(z) = w_2$. Since the optimal proposal to induce a challenge with $\zeta_2(z) < w_2$ involves equal compromise on ideology, $\epsilon_1(z)$ decreases as $(\hat{x}_1 - \hat{x}_2)^2$ increases. This proves part 3 of Proposition 1.[3] □

Proof of Proposition 2. If party 1 induces unanimity by offering $u_2(z) = u_2(s) - w_1$, its payoff is given by

$$u_1(z) = -\left(\frac{\hat{x}_2 - \hat{x}_1}{2}\right)^2 - \left(\frac{\hat{x}_1 + \hat{x}_2}{2}\right)^2 + (q - \hat{x}_2)^2 + \alpha y_1^q + w_1. \tag{E.26}$$

Suppose the parties are sufficiently distant ideologically so that party 1 prefers a

[3]Carrying out similar comparisons between other types of challenge stage equilibria yield similar results and hence are not repeated here.

challenge stage equilibrium with $\zeta_2(z) = w_2$. With the optimal proposal given by $z = (\hat{x}_1, 1, 0)$, party 1's maximum expected payoff from this challenge becomes

$$u_1(s) + \frac{w_1}{w_1 + w_2}[\alpha - u_1(s)] - w_1 \tag{E.27}$$

if $\zeta_1(z) = w_1$, and

$$u_1(s) + \left(1 - \sqrt{\frac{w_2}{\alpha - u_1(s)}}\right)[\alpha - u_1(s)] - \sqrt{w_2(\alpha - u_1(s))} + w_2 \tag{E.28}$$

if $\zeta_1(z) = \sqrt{w_2 \epsilon_1(z)} - w_2$.

Comparing E.26 first with E.27 suggests that party 1 becomes more likely to prefer a settlement over a challenge for low values of $u_2(s)$, and high values of w_1 and w_2. Comparing E.26 with E.28 confirms the relationship with $u_2(s)$ and w_1. However, differentiating E.28 with respect to w_2 indicates that higher values of w_2 make settlement more likely to be preferred only if $w_2 < \alpha - u_1(s)$.

To complete the proof, suppose that the parties are ideologically closer so that party 1 would prefer a challenge stage equilibrium with $\zeta_2(z) < w_2$. Focusing on the equilibrium with $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1 \epsilon_2(z)} - w_1)$, party 1's expected payoff from this referendum is as given in E.23, where z is such that $\epsilon_1(z) \geq \frac{w_1 \epsilon_2(z)}{\sqrt{w_1 \epsilon_2(z) - w_1}}$ and $\epsilon_2(z) \in \left(w_1, \frac{(w_1+w_2)^2}{w_1}\right)$. Comparing E.26 with E.23 confirms the above results on $u_2(s)$ and w_1. Therefore, we can conclude that a lower $u_2(s)$ and a higher w_1 unambiguously make settlement more likely to be observed. This completes the proof of Proposition 2.

$\square$

Proof of Lemma 2. The proof is an application of the main result in Baik (2008) for players with a budget constraint.

Based on 3.11, $C_Z^1(z)$ satisfies

$$\epsilon_1(z)\left[\frac{C_S(z)}{(C_Z^1(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } C_Z^1(z)>w_1+w_h\\ =0 & \text{if } C_Z^1(z)\in[0,w_1+w_h]\\ \leq 0 & \text{if } C_Z^1(z)=0.\end{cases}\tag{E.29}$$

Similarly, based on 3.12, $C_Z^h(z)$ satisfies

$$\epsilon_h(z)\left[\frac{C_S(z)}{(C_Z^h(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } C_Z^h(z)>w_1+w_h\\ =0 & \text{if } C_Z^h(z)\in[0,w_1+w_h]\\ \leq 0 & \text{if } C_Z^h(z)=0.\end{cases}\tag{E.30}$$

Accordingly, the individual campaign spending of parties 1 and h must satisfy the following first-order conditions in equilibrium for any given $C_S(z)=\zeta_j(z)$:

$$\epsilon_1(z)\left[\frac{C_S(z)}{(\zeta_1(z)+\zeta_h(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_1(z)>w_1\\ =0 & \text{if } \zeta_1(z)\in[0,w_1]\\ \leq 0 & \text{if } \zeta_1(z)=0;\end{cases}\tag{E.31}$$

$$\epsilon_h(z)\left[\frac{C_S(z)}{(\zeta_1(z)+\zeta_h(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_h(z)>w_h\\[2ex] =0 & \text{if } \zeta_h(z)\in[0,w_h]\\[2ex] \leq 0 & \text{if } \zeta_h(z)=0.\end{cases} \tag{E.32}$$

First, suppose that $C_Z^1(z)\leq w_1$ so that solving E.29 yields $C_Z^1(z)=\sqrt{\epsilon_1(z)C_S(z)}-C_S(z)$. Then, the individual best response of party 1 to its partner must also be less than or equal to w_1. Furthermore, it must equal $C_Z^1(z)$. By the assumption in Lemma 2 that $\epsilon_1(z)\geq\epsilon_h(z)$, it must be true that $C_Z^1(z)\geq C_Z^h(z)$. Therefore, the best response of party h to party 1's best response of choosing $C_Z^1(z)$ for any given $\zeta_h(z)$ is to spend a zero amount on the group's campaign. In equilibrium, simultaneous best responding implies $\zeta_1(z)=C_Z^1(z)$ and $\zeta_2(z)=0$ for a total equilibrium campaign spending of $C_Z(z)=C_Z^1(z)$. This proves part 1 of Lemma 2.

For part 2, suppose that $C_Z^h(z)\geq w_1+w_h$, which implies $C_Z^1(z)\geq w_1+w_h$. Then, the individual best response of each party to the other must be greater than its respective budget. This implies that we must have $\zeta_1(z)=w_1$ and $\zeta_h(z)=w_h$ in equilibrium, for a total campaign spending of $C_Z(z)=w_1+w_h$. This proves part 2 of Lemma 2.

For the final part of the lemma, suppose the proposal z is such that $C_Z^1(z)>w_1$ and $C_Z^h(z)\leq w_1+w_h$. First, consider the case in which $C_Z^h(z)\leq w_1$. For any $\zeta_1(z)\in[C_Z^h(z),w_1]$, party h's individual best response to party 1 is to choose a zero amount of campaign spending since $\zeta_1(z)\geq C_Z^h(z)$. This would imply a total campaign spending of $C_Z(z)\in[C_Z^h(z),w_1]$. However, since $C_Z^1(z)>w_1$, this cannot be optimal for party 1. Specifically, party 1 would have an incentive to increase its spending up to w_1. Similarly, for any $\zeta_1(z) < C_Z^h(z)$, party h best responds by choosing $\zeta_h(z) = C_Z^h(z) - \zeta_1(z)$, resulting in a total campaign spending of $C_Z(z) = C_Z^h(z)$. However, since $C_Z^1(z) > C_Z^h(z)$, this also cannot be optimal for party 1. Therefore, the only equilibrium occurs at $\zeta_1(z) = w_1$ and $\zeta_2(z) = 0$, yielding $C_Z(z) = w_1$. In this case, party 1 cannot increase its individual spending since it is already exhausting its budget and does not have an incentive to decrease it since $C_Z^1(z) > w_1$. Party h does not have an incentive to increase its spending either since $C_Z^h(z) \leq w_1$. Therefore, if $C_Z^1(z) > w_1$ and $C_Z^h(z) \leq w_1$, the equilibrium is such that $\zeta_1(z) = w_1$ and $\zeta_2(z) = 0$.

Second, consider the case in which $w_1 < C_Z^h(z) \leq w_1 + w_2$. We know that any $\zeta_1(z) < w_1$ cannot be an equilibrium, since party h would best respond to it by choosing $\zeta_h(z) = C_Z^h(z) - \zeta_1(z)$ and the resulting total campaign spending $C_Z(z) = C_Z^h(z)$ would be suboptimal from party 1's point of view. Specifically, party 1 would have an incentive to increase its spending from $\zeta_1(z) < w_1$ to w_1. Therefore, the only equilibrium is such that $\zeta_1(z) = w_1$ and $\zeta_h(z) = C_Z^h(z) - w_1$, yielding the same $C_Z(z) = C_Z^h(z)$. This completes the proof of part 3 of the lemma. □

Proof of Lemma 3. Lemma 2 characterized the optimal campaign spending of parties 1 and h within the group N_Z. For any given $C_Z(z)$, the optimal campaign spending of the only member of group N_S, party j, is such that

$$\zeta_j(z) \in \underset{C\in[0,w_j]}{\arg\max} \; \frac{C_Z(z)}{C + C_Z(z)} u_j(z) + \frac{C}{C + C_Z(z)} u_j(s) - C. \tag{E.33}$$

The first-order conditions that the optimal $\zeta_j(z)$ needs to satisfy are given by

$$\epsilon_j(z)\left(\frac{C_Z(z)}{(C_Z(z)+\zeta_j(z))^2}\right)-1\begin{cases}\geq 0 & \text{if } \zeta_j(z)>w_j\\ \\ =0 & \text{if } \zeta_j(z)\in[0,w_j]\\ \\ \leq 0 & \text{if } \zeta_j(z)=0,\end{cases} \tag{E.34}$$

where $C_Z(z) = \zeta_1(z)+\zeta_h(z)$. Note that party j cares only about $C_Z(z)$ and not about how its burden is shared among the members of group N_Z. Thus, for any given $C_Z(z)$, party j best responds by choosing a campaign spending equal to either w_j or $\sqrt{\epsilon_j(z)C_Z(z)}-C_Z(z)$, whichever is smaller.

To solve for the best response of group N_Z to any given amount of $C_S(z)$, first suppose the proposal z is such that $\epsilon_1(z)\geq\epsilon_h(z)$. Based on E.29 and E.30, the best response $C_Z(z)$ in this case is given by

$$C_Z(z)=\begin{cases}\sqrt{\epsilon_1(z)C_S(z)}-C_S(z) & \text{if } C_Z^1(z)<w_1\\ \\ w_1+w_h & \text{if } C_Z^h(z)\geq w_1+w_h\\ \\ \max\{w_1,C_Z^h(z)\} & \text{if } C_Z^1(z)>w_1 \text{ and } C_Z^h(z)\leq w_1+w_h.\end{cases} \tag{E.35}$$

On the other hand, if the proposal z is such that $\epsilon_h(z)\geq\epsilon_1(z)$, then the best response

$C_Z(z)$ to any given $C_S(z)$ becomes

$$C_Z(z) = \begin{cases} \sqrt{\epsilon_h(z)C_S(z)} - C_S(z) & \text{if } C_Z^h(z) < w_h \\ \\ w_1 + w_h & \text{if } C_Z^1(z) \geq w_1 + w_h \\ \\ \max\{w_h, C_Z^1(z)\} & \text{if } C_Z^h(z) > w_h \text{ and } C_Z^1(z) \leq w_1 + w_h. \end{cases} \tag{E.36}$$

Thus, solving for the unique pure-strategy equilibrium of the challenge stage in which both groups are simultaneously best responding to each other yields the following candidates for the equilibrium triplet $(\zeta_1(z), \zeta_h(z), \zeta_j(z))$:

1. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\epsilon_1(z) \leq \frac{(w_1+w_j)^2}{w_j}$; and $\epsilon_j(z) \geq \frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j} - w_j}$.

2. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, w_h, w_j)$ if and only if $\epsilon_1(z) \geq \frac{(\sum_k w_k)^2}{w_j}$; $\epsilon_h(z) \geq \frac{(\sum_k w_k)^2}{w_j}$; and $\epsilon_j(z) \geq \frac{(\sum_k w_k)^2}{w_1+w_h}$.

3. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, \max\{\sqrt{\epsilon_h(z)w_j} - w_j - w_1, 0\}, w_j)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\epsilon_1(z) \geq \frac{(w_1+w_j)^2}{w_j}$; $\epsilon_h(z) \leq \frac{(\sum_k w_k)^2}{w_j}$; and $\epsilon_j(z) \geq \max\{\frac{\epsilon_h(z)w_j}{\sqrt{\epsilon_h(z)w_j} - w_j}, \frac{(w_1+w_j)^2}{w_1}\}$.

4. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (0, \sqrt{\epsilon_h(z)w_j} - w_j, w_j)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\epsilon_h(z) \leq \frac{(w_h+w_j)^2}{w_h}$; and $\epsilon_j(z) \geq \frac{\epsilon_h(z)w_j}{\sqrt{\epsilon_h(z)w_j} - w_j}$.

5. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\max\{\sqrt{\epsilon_1(z)w_j} - w_j - w_h, 0\}, w_h, w_j)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\epsilon_1(z) \leq \frac{(\sum_k w_k)^2}{w_j}$; $\epsilon_h(z) \geq \frac{(w_h+w_j)^2}{w_j}$; and $\epsilon_j(z) \geq \max\{\frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j} - w_j}, \frac{(w_h+w_j)^2}{w_h}\}$.

6. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2}, 0, \frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}\right)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\left(\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2}\right) < w_1$; and $\left(\frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}\right) < w_j$.

7. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, w_h, \sqrt{\epsilon_j(z)(w_1+w_h)} - w_1 - w_h)$ if and only if $\sqrt{\bar{\epsilon}(z)} \geq \sqrt{\frac{(w_1+w_h)\epsilon_j(z)}{\bar{\epsilon}(z)}} + \sqrt{\frac{(w_1+w_h)\bar{\epsilon}(z)}{\epsilon_j(z)}}$; and $\epsilon_j(z) \leq \frac{(\sum_k w_k)^2}{w_1+w_h}$, where $\bar{\epsilon}(z) \equiv \max\{\epsilon_1(z), \epsilon_h(z)\}$.

8. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(w_1, \max\{\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} - w_1, 0\}, \max\{\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_1} - w_1\}\right)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2} > w_1$; $\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} < w_1 + w_h$; and $\max\{\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_1} - w_1\} < w_j$.

9. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(0, \frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2}, \frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}\right)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} < w_h$; and $\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2} < w_j$.

10. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(\max\{\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2} - w_h, 0\}, w_h, \max\{\frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_h} - w_h\}\right)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} > w_h$; $\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2} < w_1 + w_h$; and $\max\{\frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_h} - w_h\} < w_j$.

In each of these equilibrium candidates, it can be observed that $\zeta_k(z)$ is increasing as the value of $\epsilon_k(z)$ increases. In addition, party h free-rides on party 1's campaign spending only if the proposal z is such that $\epsilon_1(z) \geq \epsilon_h(z)$. This can be seen by inspecting the above candidates in which $\zeta_h(z) = 0$ and $\zeta_h(z) = C_Z^h(z) - w_1$. This completes the proof of Lemma 3. □

Proof of Proposition 3. Consistent with backward induction, I first focus on the acceptance strategies $a_2(z)$ and $a_3(z)$ of the non-proposer parties 2 and 3 for any given proposal z. Each party's payoff from voting to accept or reject the proposal depends on the other party's vote. First, for any given proposal z, if $a_k(z) = 1$ for both k, then each party k gets a sure payoff of $u_k(z)$. Second, if $a_2(z) = 1$ and $a_3(z) = 0$, then the bill moves to a challenge stage in which $\rho_1(z) = \rho_2(z) = Z$ and

$\rho_3(z) = S$, with the associated equilibrium campaign spending of each party given by $(\zeta_1(z), \zeta_2(z), \zeta_3(z))$. In this case, each party's receives an expected payoff determined by the specified challenge. Third, if $a_2(z) = 0$ and $a_3(z) = 1$, the challenge stage features $\rho_1(z) = \rho_3(z) = Z$ and $\rho_2(z) = S$, along with each party's associated equilibrium campaign spending. Finally, if $a_k(z) = 0$ for both parties, then each party k receives its status-quo payoff $u_k(s)$.

For any given proposal z, $a_2(z) = 1$ is a dominant strategy for party 2 if a) $u_2(z)$ is at least as great as its expected payoff from a challenge with $\rho_1(z) = \rho_3(z) = Z$ and $\rho_2(z) = S$; and b) its expected payoff from a challenge with $\rho_1(z) = \rho_2(z) = Z$ and $\rho_3(z) = S$ is at least as great as $u_2(s)$. Similarly, $a_3(z) = 1$ is a dominant strategy for party 3 if a) $u_3(z)$ is at least as great as its expected payoff from a challenge with $\rho_1(z) = \rho_2(z) = Z$ and $\rho_3(z) = S$; and b) its expected payoff from a challenge with $\rho_1(z) = \rho_3(z) = Z$ and $\rho_2(z) = S$ is at least as great as $u_3(s)$.

In order for party 1 to induce unanimity as the unique equilibrium outcome of the game, the proposal z must be such that the following conditions based on the non-proposer parties' acceptance strategies hold:[4]

- $u_k(z)$ is at least as great as party k's expected payoff from a challenge with $\rho_k(z) = S$ for $k = 2, 3$;
- The following two conditions for $k = 2, 3$ do not simultaneously hold: $u_k(s)$ is at least as great as party k's expected payoff from a challenge with $\rho_k(z) = Z$.

To solve for the optimal proposal z from party 1's perspective that would induce

[4] I again assume that a party k votes to accept a proposal z when indifferent.

unanimity in the parliament by satisfying the above conditions, we need to find the unanimity-inducing offer z for each of the possible challenge stage equilibrium candidates identified in Lemma 3 and compare party 1's unanimity payoff for all such offers.

I first focus on the first five equilibrium candidates listed in the proof of Lemma 3 in which $\zeta_j(z) = w_j$. Consider a proposal z such that $\epsilon_1(z) \leq \frac{(w_1+w_j)^2}{w_j}$, $\epsilon_1(z) \geq \epsilon_h(z)$, and $\epsilon_j(z) \geq \frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j} - w_j}$, which would give rise to a challenge stage equilibrium listed in item one if rejected. Suppose party 1 chooses $h = 2$ and $j = 3$ so that if rejected, this proposal would imply a challenge stage equilibrium with $\rho_2(z) = Z$, $\rho_3(z) = S$, and $(\zeta_1(z), \zeta_2(z), \zeta_3(z)) = (\sqrt{\epsilon_1(z)w_3} - w_3, 0, w_3)$. Party 3's expected payoff from this challenge is given by

$$\left(\frac{\sqrt{\epsilon_1(z)w_3} - w_3}{\sqrt{\epsilon_1(z)w_3}}\right) u_3(z) + \left(\frac{w_3}{\sqrt{\epsilon_1(z)w_3}}\right) u_3(s) - w_3. \tag{E.37}$$

Then, party 3 accepts this offer if and only if $u_3(z)$ is at least as great as E.37, which implies that we must have $\epsilon_3(z) \leq \sqrt{\epsilon_1(z)w_3}$.

To derive party 2's acceptance condition, suppose that party 1 now chooses $h = 3$ and $j = 2$ so that this offer goes to a challenge in which $\rho_2(z) = S$ and $\rho_3(z) = Z$. In this scenario, the equilibrium levels of campaign spending are given by $(\zeta_1(z), \zeta_2(z), \zeta_3(z)) = (\sqrt{\epsilon_1(z)w_2} - w_2, w_2, 0)$, yielding the following expected payoff for party 2:

$$\left(\frac{\sqrt{\epsilon_1(z)w_2} - w_2}{\sqrt{\epsilon_1(z)w_2}}\right) u_2(z) + \left(\frac{w_2}{\sqrt{\epsilon_1(z)w_2}}\right) u_2(s) - w_2. \tag{E.38}$$

Then, party 2 accepts this offer if and only if $\epsilon_2(z) \leq \sqrt{\epsilon_1(z)w_2}$.

In order for unanimity to be realized, the additional condition that $u_k(s)$ is not at least as great as party k's expected payoff from a challenge with $\rho_k(z) = Z$ for both $k = 2, 3$ needs to be met. To check for this, construct party k's expected payoff from a challenge with $\rho_k(z) = Z$ for $k = 2, 3$:

$$\left(\frac{\sqrt{\epsilon_1(z)w_{-k}} - w_{-k}}{\sqrt{\epsilon_1(z)w_{-k}}}\right) u_k(z) + \left(\frac{w_{-k}}{\sqrt{\epsilon_1(z)w_{-k}}}\right) u_k(s). \tag{E.39}$$

The condition that $u_k(s)$ is at least as great as E.39 reduces to $u_k(s) \geq u_k(z)$ for $k = 2, 3$. Thus, unanimity requires that $u_2(s) \geq u_2(z)$ and $u_3(s) \geq u_3(z)$ are not simultaneously true for proposal z.

First, suppose without loss of generality that the proposal z is such that $u_2(s) < u_2(z)$. Then, the conditions that need to hold for unanimity are $u_2(z) \geq u_2(s)$ and $u_3(z) \geq u_3(s) - \sqrt{\epsilon_1(z)w_3}$. Since the challenge stage equilibrium under consideration requires that $\epsilon_2(z) \leq \epsilon_1(z) \leq \frac{(w_1+w_3)^2}{w_3}$ and $\epsilon_3(z) \geq \frac{\epsilon_1(z)w_3}{\sqrt{\epsilon_1(z)w_3} - w_3}$, bringing these conditions together with the parties' acceptance criteria implies the following: Party 2 accepts any proposal z such that $u_2(z) \in [u_2(s), u_2(s) + \frac{(w_1+w_3)^2}{w_3}]$. However, there exists no proposal z that simultaneously satisfies $\epsilon_3(z) \geq \frac{\epsilon_1(z)w_3}{\sqrt{\epsilon_1(z)w_3} - w_3}$ and party 3's acceptance criteria. Second, suppose the proposal z is such that $u_3(s) < u_3(z)$. Carrying out the same analysis as above this time yields the result that party 2's acceptance criteria cannot be reconciled with the equilibrium conditions on z. Therefore, any proposal z that would imply a subsequent challenge stage equilibrium with $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$ if rejected cannot induce unanimity in the parliament.

Carrying out the same analysis for equilibrium candidates numbered two through five in the proof of Lemma 3 yields the same result as the first equilibrium candidate above. In the interest of space, each of these analyses will not be presented separately. As a result, we can conclude that any proposal z for which $\zeta_j(z) = w_j$ will be rejected by party $j \in N_S$. This proves part 1 of Proposition 3.

Consider the sixth equilibrium candidate listed in the proof of Lemma 3 in which the proposal z leads to an interior challenge equilibrium characterized by $(\zeta_1(z), \zeta_h(z), \zeta_j(z))$ $= \left(\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2}, 0, \frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}\right)$. Based on their expected payoffs, the acceptance criteria for parties $k = 2, 3$ become $\epsilon_k(z) \leq 0$. Moreover, if a proposal z meets either one of these acceptance criteria, then the final condition for achieving unanimity is also met. Therefore, if party 1 wanted to induce unanimity with a proposal z that would lead to the challenge stage equilibrium in item six if rejected, it chooses z in order to maximize $u_1(z)$ subject to the parties' acceptance criteria and the equilibrium conditions. Solving this program yields the following two alternative optimal unanimity-inducing offers: First, party 1 can choose $x = \frac{\hat{x}_1+\hat{x}_3}{2}$, thereby compromising ideologically with party 3. In addition, it can offer the following rent shares:

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right], \tag{E.40}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2\right], \tag{E.41}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right]. \tag{E.42}$$

Second, it can choose $x = \frac{\hat{x}_1+\hat{x}_2}{2}$, compromising ideologically with party 2, and offer

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.43}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.44}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2\right]. \tag{E.45}$$

Having solved for the optimal way to induce unanimity with a proposal that would induce a challenge stage equilibrium as listed in item six if rejected, now consider the seventh equilibrium candidate characterized by $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, w_h, \sqrt{\epsilon_j(z)(w_1+w_h)} - w_1 - w_h)$. Based on their expected payoffs, the acceptance criteria of parties $k = 2, 3$ become $\epsilon_k(z) \leq w_1 + w_{-k}$. Moreover, the final unanimity condition implies that we must have $\epsilon_k(z) \geq w_k\sqrt{\frac{\epsilon_{-k}(z)}{w_1+w_k}}$ for at least one $k \in \{2, 3\}$. Without loss of generality, suppose that this condition holds for party 2. Then, the unanimity conditions yield $\epsilon_3(z) \leq w_1 + w_2$ and $\epsilon_2(z) \leq w_2$. Confirming that there exist proposals z that can simultaneously satisfy these and the equilibrium conditions for item seven, party 1 maximizes $u_1(z)$ by choosing z subject to the above two constraints. Solving this program yields the following two alternative unanimity-inducing offers: First, party 1 can choose $x = \frac{\hat{x}_1+\hat{x}_3}{2}$ and offer the following rent shares:

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + w_1 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right], \tag{E.46}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q + w_2 - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2\right], \tag{E.47}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (w_1+w_2) - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right]. \tag{E.48}$$

Second, party 1 can choose $x = \frac{\hat{x}_1+\hat{x}_2}{2}$ and offer

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + w_1 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.49}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (w_1+w_3) - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.50}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q + w_3 - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2\right]. \tag{E.51}$$

Now consider the eighth equilibrium candidate characterized by $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \Big(w_1, \max\{\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} - w_1, 0\}, \max\{\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_1} - w_1\}\Big)$. A similar analysis suggests that there again exist two unanimity-inducing offers corresponding to two different acceptance criteria: First, $u_2(z) \geq u_2(s) - w_1$ and $u_3(z) \geq u_3(s)$; and second $u_2(z) \geq u_2(s)$ and $u_3(z) \geq u_3(s) - w_1$. Checking that there exist proposals z that satisfy both the acceptance criteria and the equilibrium conditions, we can proceed with party 1's maximization problem. If party 1 chooses a unanimity-inducing offer z based on the first acceptance criteria, the offer z involves $x = \frac{\hat{x}_1+\hat{x}_3}{2}$, y_1 as given in E.46, and

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - w_1 - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2\right], \tag{E.52}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right]. \tag{E.53}$$

On the other hand, if it chooses the offer based on the second acceptance criteria, the offer z now involves $x = \frac{\hat{x}_1+\hat{x}_2}{2}$, y_1 as given in E.49, and

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.54}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - w_1 - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2\right]. \tag{E.55}$$

The ninth equilibrium candidate is similar to the sixth candidate in the sense that the partners completely free-ride in both cases and the groups fight unconstrained against each other. The only difference is the identity of the partner. Thus, partner party h's acceptance criteria is stricter, requiring a higher premium from party 1. Thus, this way to induce unanimity will never be optimal for party 1.

Finally, consider the tenth equilibrium candidate, which implies the following alternative acceptance criteria: First, $u_3(z) \geq u_3(s)$ and $u_2(z) \geq u_2(s) + w_2$, and second $u_3(z) \geq u_3(s)+w_3$ and $u_2(z) \geq u_2(s)$. If party 1 chooses a unanimity-inducing offer z based on the first acceptance criteria, the offer z involves $x = \frac{\hat{x}_1+\hat{x}_3}{2}$,

$$y_1 = \alpha^{-1}\left[\alpha y_1^q - w_2 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right], \tag{E.56}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q + w_2 - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2\right], \tag{E.57}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right]. \tag{E.58}$$

On the other hand, if it chooses this offer based on the second acceptance criteria, then the offer z involves $x = \frac{\hat{x}_1+\hat{x}_2}{2}$,

$$y_1 = \alpha^{-1}\left[\alpha y_1^q - w_3 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.59}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{E.60}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q + w_3 - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2\right]. \tag{E.61}$$

The above analysis indicates that the utility each party settles for in a grand bargain reflects its strength in the post-bargaining stage. To see this, first consider equilibrium candidate seven in which both members of group N_Z fight against N_S with all their resources. In this case, the two parties who would belong to N_Z if the proposal z is rejected can each extract a premium equal to their campaigning budgets from the party that would belong to N_S in a grand bargain. In the equilibrium candidate eight, the non-proposer partner party is at least partially free-riding on party 1's campaign spending. Thus, party 1 is able to extract from its partner an amount equal to its campaigning budget when inducing a settlement. Equilibrium candidate ten demonstrates the reverse of this situation with party 1 free-riding on its partner's campaign spending. This proves part 2 of Proposition 3.

Part 3 of the proposition describes the optimal way for the proposer to induce unanimity. Comparing the maximum value of $u_1(z)$ from a unanimous agreement in each of the cases considered above, it can be observed that party 1 can secure the maximum payoff from unanimity with a proposal z that satisfies the equilibrium conditions of items seven or eight. Although the optimal z that induces unanimity in these two cases is different, they both imply the same sure-payoff for party 1. Specifically, for each of these cases, party 1 can induce unanimity by proposing either $x = \frac{\hat{x}_1+\hat{x}_3}{2}$ and E.46 for itself, or $x = \frac{\hat{x}_1+\hat{x}_2}{2}$ and E.49 for itself. Its rent share in either of these cases indicates that it is increasing in y_1^q, w_1, $(q-\hat{x}_2)$, and $(q-\hat{x}_3)$. Moreover, since each party gets compensated for their ideological utility loss in the grand bargain through its rent share as can be observed in E.47, E.48, E.50, and E.51, party 1's unanimity payoff strictly increases as the three parties get ideologically closer. This completes the proof of Proposition 3. □

Proof of Proposition 4. In order to analyze the optimal proposals to get to a given challenge stage equilibrium for party 1, I first focus on the non-proposer parties' voting strategies. In order for a proposal z to induce a unique challenge stage equilibrium with $\rho_h(z) = Z$ and $\rho_j(z) = S$ for $h, j \in \{2, 3\}$ and $h \neq j$, the following conditions must hold:

- Party h's expected payoff from a challenge with $\rho_h(z) = Z$ must be at least as great as $u_h(s)$;
- Party j's expected payoff from a challenge with $\rho_j(z) = S$ must be at least as great as $u_j(z)$;
- The following conditions do not simultaneously hold: Party h's expected payoff from a challenge with $\rho_h(z) = S$ is at least as great as $u_h(z)$; and party j's expected payoff from a challenge with $\rho_j(z) = Z$ is at least as great as $u_j(s)$.

Consider the challenge stage equilibrium candidate listed in item one in the proof of

Lemma 3. In order to induce a challenge stage equilibrium with $\rho_h(z) = Z$, $\rho_j(z) = S$, and $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$, party 1's proposal z must meet the corresponding equilibrium conditions, satisfy party h's acceptance criteria, and violate party j's acceptance criteria. The analysis in Proposition 3 indicated that party j will reject any offer z that would give rise to this equilibrium if rejected. In addition, among the range of proposals that would give rise to this challenge if rejected in the parliament, party h will accept any z such that $u_h(z) \in [u_h(s), u_h(s) + \frac{(w_1+w_j)^2}{w_j}]$.

In equilibrium, party 1 will not offer any higher surplus to party h than is required to get its acceptance. Thus, the optimal z to induce this challenge will be such that $u_h(z) = u_h(s)$. Moreover, since $\zeta_j(z) = w_j$ for any proposal z in this range, party 1 cannot influence the amount of $C_S(z)$. Thus, the proposal z need not worry about party j's rejection as long as it satisfies the equilibrium conditions. The Lagrangian of this problem can be written as

$$L = -(x-\hat{x}_1)^2+y_1-2\sqrt{w_j\epsilon_1(z)}+w_j+(\lambda_1-\lambda_2)[-(x-\hat{x}_h)^2+1-y_1-u_h(s)]+\lambda_2\frac{(w_1+w_j)^2}{w_j}$$

$$+\lambda_3\left[-(x-\hat{x}_j)^2 - \frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j} - w_j}\right]. \tag{E.62}$$

With $y_j = 0$, solving this program for x, y_1, and $y_h = 1 - y_1$ yields $x = \frac{\hat{x}_1+\hat{x}_h}{2}$,

$$y_1 = y_1^q + y_j^q + (q - \hat{x}_h)^2 - \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2, \tag{E.63}$$

$$y_h = y_h^q - (q - \hat{x}_h)^2 + \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2. \tag{E.64}$$

Proceeding with a similar analysis for the remaining equilibrium candidates yields the result that the optimal proposal z to induce any challenge stage equilibrium involves offering $x = \frac{\hat{x}_1+\hat{x}_h}{2}$. Since party h requires at least $u_h(s)$ in order to become party 1's partner in a challenge regardless of how much it will spend, it can be observed from E.63 and E.64 that party 1's winning prize increases as y_h^q decreases, $(q-\hat{x}_h)^2$ increases, and it gets ideologically closer to party h. Moreover, since $\epsilon_h(z)$ increases as $u_h(s)$ decreases for any proposal z, Lemma 3 indicates that $\zeta_h(z)$ would be weakly higher, thus weakly increasing the proposal's winning probability. Therefore, party 1's expected payoff would increase. This proves part 1 of Proposition 4.

Part 2 of Proposition 4 is concerned with how party 1's expected payoff from a challenge is affected by its partner's campaigning budget. In the interest of brevity, I do not present here the solutions for the optimal proposals that would induce each possible challenge stage equilibrium. Instead, I focus on two examples that demonstrate party 1's different incentives with regards to the other parties' campaigning budgets.

Consider the equilibrium candidate listed in item two in the proof of Lemma 3. Solving for the optimal proposal to induce this particular challenge equilibrium yields $x = \frac{\hat{x}_1+\hat{x}_h}{2}$, $y_j = 0$,

$$y_1 = y_1^q + y_j^q + (q-\hat{x}_h)^2 - \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2 - \frac{(w_1+w_h+w_j)^2}{w_j}, \tag{E.65}$$

$$y_h = y_h^q - (q-\hat{x}_h)^2 + \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2 + \frac{(w_1+w_h+w_j)^2}{w_j}. \tag{E.66}$$

As a result, party 1's maximum expected payoff from this type of challenge becomes

$$\left(\frac{w_1 + w_h}{w_1 + w_h + w_j}\right)\left[y_j^q + \sum_{k=1,h}(q - \hat{x}_k)^2 - 2\left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2 - \frac{(w_1 + w_h + w_j)^2}{w_j}\right] + u_1(s) - w_1. \tag{E.67}$$

Differentiating E.67 with respect to w_h yields

$$\frac{(w_j)^2 \epsilon_1(z) - 2(w_1 + w_h)(w_1 + w_h + w_j)}{(w_1 + w_h + w_j)^2 w_j} \tag{E.68}$$

where $\epsilon_1(z)$ is calculated using the optimal proposal and equals the expression in brackets in E.67. The sign of this expression depends on the parameters of the model. Specifically, it is negative if

$$\epsilon_1(z) < \frac{2(w_1 + w_h)(w_1 + w_h + w_j)}{(w_j)^2}, \tag{E.69}$$

and positive otherwise. Thus, we conclude that a higher w_h decreases party 1's expected payoff from the type of challenge in item 2 of Lemma 3 if $\epsilon_1(z)$ is sufficiently small, which happens if $u_1(s)$ is large, or if w_1 or w_h are high. Analyzing other equilibrium candidates in which $\zeta_h(z) > 0$ indicates that this relationship holds more generally. This proves part 2 of Proposition 4.

For Part 3, consider party 1's decision on the identity of party h. Since a lower $u_h(s)$ and higher $u_j(s)$ necessarily increase party 1's expected payoff in any challenge equilibrium, it follows that holding everything else constant, party 1 would prefer to partner with the party that commands the lower status-quo payoff.

To see how the partner decision is affected by the parties' campaigning budgets, consider an alternative challenge equilibrium in which the proposal z is such that $\zeta_h(z) = 0$. Since it has already been analyzed, I focus on the equilibrium given in the first item in Lemma 3. With the proposal z given by $x = \frac{\hat{x}_1+\hat{x}_h}{2}$, $y_j = 0$, y_1 as in E.63, and y_h as in E.64, party 1's maximum expected payoff from this challenge becomes

$$\left[1 - \sqrt{\frac{w_j}{\epsilon_1(z)}}\right] \epsilon_1(z) + u_1(s), \tag{E.70}$$

where $\epsilon_1(z) = -2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 + y_j^q + (q - \hat{x}_1)^2 + (q - \hat{x}_h)^2$. It can be observed from E.70 that it does not depend on w_h and depends negatively on w_j. Furthermore, this relationship holds in other challenge equilibrium candidates in which $\zeta_h(z) = 0$. Thus, party 1 would prefer to have as its opponent the party with the lower campaigning budget in such challenge equilibria. The rest of the proposition follows from the analysis for part 2. Thus, this completes the proof of Proposition 4. □

Proof of Proposition 5. Analyzing party 1's incentives between inducing a grand bargain and a challenge requires comparing its maximum payoff from each of the two outcomes. However, since the type of challenge equilibrium that will maximize party 1's expected payoff depends on different conditions on the parameters of the model, I only present here the relevant results from comparing party 1's maximum unanimity payoff with certain types of challenge stage equilibria for the sake of brevity.

First, consider the challenge stage equilibrium listed in item one in the proof of Lemma 3, where $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$. Given the optimal proposal z characterized in the proof of Proposition 4 that would give rise to this challenge equilibrium, party 1's maximized payoff from this challenge is as given in E.70, where $\epsilon_1(z) = -2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 + y_j^q + (q-\hat{x}_1)^2 + (q-\hat{x}_h)^2$. The proof of Proposition 3 characterized the optimal proposal z to induce unanimity, which involves $x = \frac{\hat{x}_1+\hat{x}_j}{2}$ and

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + w_1 + (q-\hat{x}_h)^2 + (q-\hat{x}_j)^2 - \left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2\right].$$

Therefore, party 1's maximum payoff from unanimity is given by

$$\alpha y_1^q + w_1 + \sum_{k=2,3}(q-\hat{x}_k)^2 - \left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2 - 2\left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2. \tag{E.71}$$

Comparing E.71 with the maximum expected payoff from the considered challenge indicates that party 1 prefers a grand bargain over this challenge if

$$w_1 - \left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2 - 2\left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2 \geq -2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 + y_j^q - \sqrt{w_j\epsilon_1(z)} - (q-\hat{x}_1)^2, \tag{E.72}$$

where $\epsilon_1(z)$ is given as before. Condition E.72 is more likely to hold if the non-proposer parties h and j each commands a lower status-quo payoff. Moreover, this relationship carries over to other types of challenge equilibria. This proves Part 1 of Proposition 5.

Notice that condition E.72 becomes more likely to hold as

$$-\left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2-2\left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2+2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 \tag{E.73}$$

increases, which, when manipulated, suggests that all parties need to be ideologically close for unanimity to be preferred. This is also a relationship that carries over to other types of challenge equilibria. This proves Part 2 of Proposition 5.

Proposition 4 indicated that the individual roles w_h and w_j might play on party 1's incentives between a grand bargain and a challenge are ambiguous and depend on the particular challenge equilibrium considered. However, to see how party 1's incentives respond to the *relative* budgets of the non-proposer parties, consider a challenge equilibrium such as item five in the proof of Lemma 3 in which $(\zeta_1(z),\zeta_h(z),\zeta_j(z)) = (\max\{\sqrt{\epsilon_1(z)w_j}-w_j-w_h,0\},w_h,w_j)$. Solving for the optimal proposal z that would lead to this challenge yields $x=\frac{\hat{x}_1+\hat{x}_h}{2}$, $y_j=0$,

$$y_1=y_1^q+(q-\hat{x}_h)^2-\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2-\frac{(w_h+w_j)^2}{w_j}, \tag{E.74}$$

$$y_h=y_h^q-(q-\hat{x}_h)^2+\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2+\frac{(w_h+w_j)^2}{w_j}. \tag{E.75}$$

The condition obtained by comparing the maximum expected payoff from this challenge and E.71 is more likely to hold as $\frac{w_h}{w_j}$ increases. Note that if $w_h > w_j$, this would require the two parameters to diverge, whereas if $w_h < w_j$, they must become more similar. However, since this is a challenge equilibrium in which the low-budget party is more likely to become the partner based on Proposition 4, it is more likely that $w_h < w_j$. Thus, more similar budgets decrease the payoff from this challenge.

This completes the proof of Proposition 5. $\square$

Bibliography

[1] Acemoglu, D. and J. Robinson, (2000), "Why Did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective," Quarterly Journal of Economics, vol. 115(4), 1167-99.

[2] Acemoglu, D. and J. Robinson, (2001), "A Theory of Political Transitions," American Economic Review, vol. 91(4), 938-63.

[3] Acemoglu, D. and J. Robinson, (2008), "Persistence of Power, Elites, and Institutions," American Economic Review, vol. 98(1), 267-93.

[4] Acemoglu, D., S. Johnson, J. Robinson and P. Yared, (2008), "Income and Democracy," American Economic Review, vol. 98(3), 808-42.

[5] Acemoglu, D., G. Egorov and K. Sonin, (2010), "Political Selection and Persistence of Bad Governments," Quarterly Journal of Economics, vol. 125(4), 1511-75.

[6] Acemoglu, D., D. Ticchi and A. Vindigni, (2011), "Emergence and Persistence of Inefficient States," Journal of the European Economic Association, vol. 9(2), 177-208.

[7] Acemoglu, D., G. Egorov and K. Sonin, (2012), "Dynamics and Stability of Constitutions, Coalitions, and Clubs," American Economic Review, vol. 102(4), 1446-76.

[8] Acemoglu, D., J. Robinson and R. Torvik, (2013), "Why Do Voters Dismantle Checks and Balances?," Review of Economic Studies, vol. 80(3), 845-75.

[9] Aghion, P., A. Alesina and F. Trebbi, (2004), "Endogenous Political Institutions," Quarterly Journal of Economics, vol. 119(2), 565-611.

[10] Alesina, A. and G. Tabellini, (1990), "A Positive Theory of Fiscal Deficits and Government Debt," Review of Economic Studies, vol. 57(3), 403-14.

[11] Ashworth, S., (2006), "Campaign Finance and Voter Welfare with Entrenched Incumbents," American Political Science Review, vol. 100(1), 55-68.

[12] Austen-Smith, D. and J. Banks, (1988), "Elections, Coalitions, and Legislative Outcomes," American Political Science Review, vol. 82(2), 405-422.

[13] Azzimonti, M., (2014), "The Dynamics of Public Investment Under Persistent Electoral Advantage," Review of Economic Dynamics, *forthcoming.*

[14] Bai, J. and R. Lagunoff, (2011), "On the Faustian Dynamics of Policy and Political Power," Review of Economic Studies, vol. 78, 17-48.

[15] Baik, K., (2008), "Contests With Group-Specific Public Good Prizes," Social Choice and Welfare, vol. 30(1), 103-17.

[16] Banks, J., (2000), "Buying Supermajorities in Finite Legislatures," American Political Science Review, vol. 94(3), 677-81.

[17] Banks, J. and J. Duggan, (2000), "A Bargaining Model of Collective Choice," American Political Science Review, vol. 94(1), 73-88.

[18] Barbera, S. and M. Jackson, (2004), "Choosing How to Choose: Self-stable Majority Rules and Constitutions," Quarterly Journal of Economics, vol. 119(3), 1011-48.

[19] Baron, D. and J. Ferejohn, (1989), "Bargaining in Legislatures," American Political Science Review, vol. 83(4), 1181-1206.

[20] Baron, D., (1994), "Electoral Competition with Informed and Uninformed Voters," American Political Science Review, vol. 88(1), 33-47.

[21] Barro, R., (1999), "Determinants of Democracy," Journal of Political Economy, vol. 107(6), 158-83.

[22] Battaglini, M. and S. Coate, (2007), "Inefficiency in Legislative Policymaking: A Dynamic Analysis," American Economic Review, vol. 97(1), 118-49.

[23] Battaglini, M., S. Nunnari and T. Palfrey, (2012), "Legislative Bargaining and the Dynamics of Public Investment," American Political Science Review, vol. 106(2), 407-29.

[24] Besley, T. and S. Coate, (1997), "An Economic Model of Representative Democracy," Quarterly Journal of Economics, vol. 112(1), 85-114.

[25] Besley, T. and S. Coate, (1998), "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis," American Economic Review, vol. 88(1), 139-56.

[26] Besley, T. and T. Persson, *Pillars of Prosperity: Political Economics of Development Clusters*, Princeton, NJ: Princeton University Press, 2011.

[27] Besley, T. and T. Persson, "Taxation and Development," Auerbach, A, R. Chetty, M. Feldstein, and E. Saez (eds) in *Handbook of Public Economics*, 2013.

[28] Besley, T., E. Ilzetzki and T. Persson, (2013), "Weak States and Steady States: The Dynamics of Fiscal Capacity," American Economic Journal: Macroeconomics, vol. 5(4), 205-35.

[29] Besley, T., T. Persson and M. Reynal-Querol, (2014), "Resilient Leaders and Institutional Reform: Theory and Evidence," Working Paper.

[30] Bowen, R., Y. Chen and H. Eraslan, (2014), "Mandatory versus Discretionary Spending: The Status Quo Effect," American Economic Review, *forthcoming.*

[31] Bowler, S. and T. Donovan, *Demanding Choices: Opinion, Voting, and Direct Democracy*, Ann Arbor, MI: University of Michigan Press, 1998.

[32] Broder, D., *Democracy Derailed: Initiative Campaigns and the Power of Money*, New York, NY: Harcourt, 2000.

[33] Bruckner, M., A. Ciccone and A. Tesei, (2012), "Oil Price Shocks, Income, and Democracy," Review of Economics and Statistics, vol. 94(2), 389-99.

[34] Buchanan, J. and G. Tullock, *Calculus of Consent: Logical Foundations of Constitutional Democracy*, Indianapolis, IN: Liberty Fund, Inc., 1962.

[35] Bueno de Mesquita, B., J. Morrow, R. Siverson and A. Smith, *The Logic of Political Survival*, Cambridge, MA: Cambridge University Press, 2003.

[36] Che, Y., Y. Lu, Z. Tao and P. Wang, (2012), "The Impact of Income on Democracy Revisited," Journal of Comparative Economics, vol. 41(1), 159-69.

[37] Coate, S., (2004), "Pareto-Improving Campaign Finance Policy," American Economic Review, vol. 94(3), 628-55.

[38] Deacon, R., (2009), "Public Good Provision under Dictatorship and Democracy," Public Choice, vol. 139, 241-62.

[39] Diermeier, D. and T. Feddersen, (1998), "Cohesion in Legislatures and the Vote of Confidence Procedure," The American Political Science Review, vol. 92(3), 611-21.

[40] Diermeier, D., H. Eraslan and A. Merlo, (2002), "Coalition Governments and Comparative Constitutional Design," Euopean Economic Review, vol. 46, 893-907.

[41] Dixit, A., (1987), "Strategic Behavior in Contests," American Economic Review, vol. 77(5), 891-98.

[42] Downs, A., *An Economic Theory of Democracy*, New York, NY: Harper and Row, 1957.

[43] Duggan, J. and T. Kalandrakis, (2012), “Dynamic Legislative Policy Making,” Journal of Economic Theory, vol. 147(5), 1653-88.

[44] Eraslan, H., (2002), “Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model,” Journal of Economic Theory, vol. 103(1), 11-30.

[45] Eraslan, H. and A. Merlo, (2002), “Majority Rule in a Stochastic Model of Bargaining,” Journal of Economic Theory, vol. 103(1), 31-48.

[46] de Figueiredo, J., C. Ji and T. Kousser, (2011), “Financing Direct Democracy: Revisiting the Research on Campaign Spending and Citizen Initiatives,” Journal of Law, Economics, and Organization, vol. 27(3), 485-514.

[47] Gerber, E., *The Populist Paradox: Interest Group Influence and the Promise of Direct Legislation*, Princeton, NJ: Princeton University Press, 1999.

[48] Goreclose, T. and J. Snyder, (1996), “Buying Supermajorities,” The American Political Science Review, vol. 90(2), 303-15.

[49] Haller, H. and S. Holden, (1997), “Ratification Requirement and Bargaining Power,” International Economic Review, vol. 38(4), 825-51.

[50] Humphreys, M., (2007), “Strategic Ratification,” Public Choice, vol. 132(1-2), 191-208.

[51] Haller, H. and R. Lagunoff, (2000), “Genericity and Markovian Behavior in Stochastic Games,” Econometrica, vol. 68(5), 1231-48.

[52] Harrington, J., (1990), "The Power of the Proposal Maker in a Model of Endogenous Agenda Formation," Public Choice, vol. 64(1), 1-20.

[53] Hassler, J., K. Storesletten and F. Zilibotti, (2007), "Democratic Public Good Provision," Journal of Economic Theory, vol. 133(1), 127-51.

[54] Hillman, A. and J. Riley, (1989), "Politically Contestable Rents and Transfers," Economics and Politics, vol. 1(1), 17-39.

[55] Hobsbawm, E., *The Age of Extremes: The Short Twentieth Century, 1914-1991*, UK: Vintage Pubishers, 1994.

[56] Horst, U., (2005), "Stationary Equilibria in Discounted Stochastic Games with Weakly Interacting Players," Games and Economic Behavior, vol. 51(1), 83-108.

[57] Huntington, S., *The Third Wave: Democratization in the Late 20th Century*, Norman, OK: University of Oklahoma Press, 1991.

[58] Iida, K., (1996), "Involuntary Defection in Two-Level Games," Public Choice, vol. 89(3-4), 283-303.

[59] Ingberman, D., (1985), "Running Against the Status-quo: Institutions for Direct Democracy Referenda and Allocations Over Time," Public Choice, vol. 46(1), 19-43.

[60] Jack, W. and R. Lagunoff, (2006), "Dynamic Enfranchisement," Journal of Public Economics, vol. 90(4-5), 551-72.

[61] Jones, B. and B. Olken, (2009), "Hit or Miss? The Effect of Assassinations on Institutions and War," American Economic Journal: Macroeconomics, vol. 1(2), 55-87.

[62] Kalandrakis, T., (2004), "A Three-Player Dynamic Majoritarian Bargaining Game," Journal of Economic Theory, vol. 116(2), 294-322.

[63] Kalandrakis, T., (2006), "Proposal Rights and Political Power," American Journal of Political Science, vol. 50(2), 441-48.

[64] Klein, P., P. Krusell and J. Rios-Rull, (2008), "Time-Consistent Public Policy," Review of Economic Studies, vol. 75(3), 789-808.

[65] Krusell, P., B. Kuruscu and A. Smith, (2002), "Equilibrium Welfare and Government Policy with Quasi-Geometric Discounting," Journal of Economic Theory, vol. 105(1), 42-72.

[66] Lagunoff, R., (2001), "A Theory of Constitutional Standards and Civil Liberties," Review of Economic Studies, vol. 68(1), 109-32.

[67] Lagunoff, R., (2008), "Markov Equilibirum in Models of Dynamic Endogenous Political Institutions," Working Paper, Georgetown University.

[68] Lagunoff, R., (2009), "Dynamic Stability and Reform of Political Institutions," Games and Economic Behavior, vol. 67(2), 569-83.

[69] Lake, D. and M. Baum, (2001), "The Invisible Hand of Democracy: Political Control and the Provision of Public Services," Comparative Political Studies, vol. 34(6), 587-621.

[70] Lipset, S., (1959), "Some Social Requisites of Democracy: Economic Development and Political Legitimacy," American Political Science Review, vol. 53(1), 69-105.

[71] Lizzeri, A. and N. Persico, (2001), "The Provision of Public Goods under Alternative Electoral Incentives," American Economic Review, vol. 91(1), 225-39.

[72] Lizzeri, A. and N. Persico, (2004), "Why Did the Elites Extend the Suffrage? Democracy and the Scope of Government, with an Application to Britain's Age of Reform," Quarterly Journal of Economics, vol. 119(2), 707-65.

[73] Lott, J., (1999), "Public Schooling, Indoctrination, and Totalitarianism," Journal of Political Economy, vol. 107(6), 127-57.

[74] Lupia, A., (1992), "Busy Voters, Agenda Control, and the Power of Information," The American Political Science Review, vol. 86(2), 390-403.

[75] Lupia, A. and J. Matsusaka, (2004), "Direct Democracy: New Approaches to Old Questions," Annual Review of Political Science, vol. 7, 463-82.

[76] Maskin, E. and J. Tirole, (2004), "The Politician and the Judge: Accountability in Government," American Economic Review, vol. 94(4), 1034-54.

[77] Matsusaka, J., (2005a), "Direct Democracy Works," Journal of Economic Perspectives, vol. 19(2), 185-206.

[78] Matsusaka, J., (2005b), "The Eclipse of Legislatures: Direct Democracy in the 21st Century," Public Choice, vol. 124(1), 157-77.

[79] McCarty, N., (2000), "Proposal Rights, Veto Rights and Political Bargaining," American Journal of Political Science, vol. 44(3), 506-22.

[80] Milesi-Ferretti, G., R. Perotti and M. Rostagno, (2002), "Electoral Systems and Public Spending," Quarterly Journal of Economics, vol. 117(2), 609-57.

[81] Persson, T. and E. O. Svensson, (1989), "Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences," Quarterly Journal of Economics, vol. 104(2), 325-45.

[82] Persson, T. and G. Tabellini, *Political Economics: Explaining Economic Policy*, Cambridge, MA: The MIT Press, 2001.

[83] Persson, T. and G. Tabellini, (2009), "Democratic Capital: The Nexus of Political and Economic Change," American Economic Journal: Macroeconomics, vol. 1(2), 88-126.

[84] Powell, R., (1996), "Bargaining in the Shadow of Power," Games and Economic Behavior, vol. 15(2), 255-89.

[85] Prezeworski, A., M. Alvarez, J.A. Cheibub and F. Limongi, *Democracy and Development: Political Institutions and Well-Being in the World, 1950-1990*, Cambridge, UK: Cambridge University Press, 2000.

[86] Putnam, R., (1988), "Diplomacy and Domestic Politics: The Logic of Two-Level Games," International Organization, vol. 42(3), 427-60.

[87] Robinson, J. and R. Torvik, (2013), "Endogenous Presidentialism," Working Paper.

[88] Romer, T. and H. Rosenthal, (1978), "Political Resource Allocation, Controlled Agendas, and the Status-quo," Public Choice, vol. 33(4), 27-43.

[89] Romer, T. and H. Rosenthal, (1979), "Bureaucrats versus Voters: On the Political Economy of Resource Allocation by Direct Democracy," Quarterly Journal of Economics, vol. 93(4), 563-587.

[90] Skaperdas, S., (1996), Contest Success Functions," Economic Theory, vol. 7(2), 283-90.

[91] Skaperdas, S. and S. Vaidya, (2012), "Persuasion as a Contest," Economic Theory, vol. 51(2), 465-86.

[92] Snyder, J., (1989), "Election Goals and the Allocation of Campaign Resources," Econometrica, vol. 57(3), 637-60.

[93] Snyder, J., M. Ting and S. Ansolabehere, (2005), "Legislative Bargaining under Weighted Voting," American Economic Review, vol. 95(4), 981-1004.

[94] Song, Z., K. Storesletten and F. Zilibotti, (2012), "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt," Econometrica, vol. 80(6), 2785-803.

[95] Szidarovszky, F. and K. Okuguchi, (1997), "On the Existence and Uniqueness of Pure Nash Equilibrium in Rent-Seeking Games," Games and Economic Behavior, vol. 18(1), 135-40.

[96] Ticchi, D. and A. Vindigni, (2010), "Endogenous Constitutions," Economic Journal, Royal Economic Society, vol. 120(3), 1-39.

[97] Tullock, G., "Efficient Rent-Seeking," in *J.M. Buchanan, R.D. Tollison and G. Tullock, Toward a Theory of the Rent-Seeking Society*, College Station, TX: Texas A.&M. University Press, 97-112, 1980.

[98] Winter, E., (1996), "Voting and Vetoing," American Political Science Review, vol. 90(4), 813-23.

[99] Wittman, D., (1989), "Why Democracies Produce Efficient Results," Journal of Political Economy, vol. 97(6), 1395-424.

[100] Yildirim, H., (2007), "Proposal Power and Majority Rule in Multilateral Bargaining with Costly Recognition," Journal of Economic Theory, vol. 136(1), 167-96.

Vita

Leyla Derin Karakaş was born on August 11, 1986 in Istanbul, Turkey. She graduated from Robert College in Istanbul in 2004 and obtained her B.S. degree in 2008 with honors from the University of Virginia's McIntire School of Commerce with a concentration in Finance and a major in Economics. She started the Economics Ph.D. program at Johns Hopkins University in August 2008. During her time there, she twice taught the undergraduate course on game theory. In 2013, she won the Dean's Teaching Fellowship that allowed her to design and teach a course on political economy. Leyla Derin Karakaş will be starting her academic career in August 2014 as an Assistant Professor of Economics at Syracuse University's Maxwell School of Citizenship and Public Affairs.